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A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED
WITH THE CO-OPERATION OF THE

AMERICAN PHYSICAL SOCIETY

BY

EDWARD L. NICHOLS

ERNEST MERRITT, and FREDERICK BEDELL

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PHYSICAL REVIEW.

VECTOR REPRESENTATION OF NON-HARMONIC ALTERNATING CURRENTS.

By Bunroku Arakawa.

PROBLEMS concerning the general theory of alternating currents are usually treated by assuming that electromotive forces and currents have wave forms that can be represented by sine curves, or they are assumed to be simple harmonic alternating quantities. But, in actual cases, the wave forms are more or less distorted from the sine curve, and some phenomena of such non-harmonic alternating quantities will be different from the corresponding phenomena produced by simple harmonic alternating quantities of equal effective values. So in these cases the necessity arises of investigating the effects upon those phenomena of wave forms of the current or electromotive force.

When the currents and electromotive forces are of simple harmonic wave form they can be represented in a well-known manner by radius-vectors, and their relations can be expressed completely by a vector diagram which in many cases greatly facilitates the solution of problems. For non-harmonic alternating currents, however, the same method of vector representation will not be generally applicable without certain modifications.

A few years ago Bedell and Tuttle[1] showed that if the electromotive force is sinusoidal and the current is non-harmonic, the higher harmonics in the current wave may be represented, collectively, by a radius-vector which is normal to radius-vectors representing the

[1] Transactions, A. I. E. E., Vol. XXV., p. 615.

electromotive force and the fundamental wave of the current, so that the whole diagram will become a space figure. They also showed that if the higher harmonics in the current wave are to be represented separately a diagram of more than three dimensions will be required, which, therefore, cannot be shown as a geometrical figure. The purpose of the present writer is to study more fully the general method by which non-harmonic alternating quantities can be represented by radius-vectors.

Since the wave form of simple harmonic alternating quantities is a definite one, such quantities are completely specified by their effective values (*i. e.*, their magnitudes) and their phase-positions, so that they can be completely represented by radius-vectors which have magnitude and direction. If, however, the alternating quantities are non-harmonic, their effective values and phase positions are not sufficient to specify them completely, and their wave forms must be given in addition. The wave form of a non-harmonic alternating quantity is determined by the magnitudes and the phase positions of higher harmonics contained in the wave. Hence if, in addition to the fundamental, only one harmonic be present, we must have four independent factors to specify a non-harmonic alternating quantity completely; therefore any method of vector representation cannot be theoretically correct, but is artificial and conventional. This is true of the method developed in this paper. While the writer does not believe this method to be simple enough for solving every practical problem of non-harmonic alternating currents, he believes it may give some help in the better understanding of the subject.

In connection with this subject consideration will also be given to the meaning of the terms "power-factor" and "reactive-factor" of a circuit carrying non-harmonic alternating currents.

Mean Value of Product of Two Alternating Quantities; Equivalent Phase Difference. — Let the instantaneous values of electromotive force and current be expressed by

$$e = E_m\{\sin \omega t + h_3 \sin (3\omega t + \theta_3) + \cdots\},$$

and

$$i = I_m\{\sin (\omega t - \varphi_1) + k_3 \sin (3\omega t + \theta_3 - \varphi_3) + \cdots\},$$

in which ω is 2π times the fundamental frequency, and h_3 and k_3 are

the amplitudes (hereafter referred to as the percentage amplitudes) of the third harmonic of electromotive force and current, respectively, the amplitudes of the fundamental being taken as unity. The phase difference between current and electromotive force is φ_1 for the fundamental and φ_3 for the third harmonic. θ_3, etc., indicate the phase positions of the harmonics in the electromotive force wave with reference to the fundamental.

Let E and I be the effective values of electromotive force and current, respectively; then, as is well known,

$$E = \frac{E_m}{\sqrt{2}}(1 + h_3^2 + \cdots)^{\frac{1}{2}},$$

$$I = \frac{I_m}{\sqrt{2}}(1 + k_3^2 + \cdots)^{\frac{1}{2}}.$$

If e and i are multiplied together, the mean value W of the product during a complete period of the fundamental wave will be found to be

$$W = \tfrac{1}{2}E_m I_m(\cos\varphi_1 + h_3 k_3 \cos\varphi_3 + \cdots);$$

or,

$$W = EI\left\{\frac{\cos\varphi_1 + h_3 k_3 \cos\varphi_3 + \cdots}{(1 + h_3^2 + \cdots)^{\frac{1}{2}}(1 + k_3^2 + \cdots)^{\frac{1}{2}}}\right\}.$$

But W is the power of the circuit, and consequently the fraction within the brackets is by definition the power-factor of the circuit. Hence, if the power-factor p is denoted by cos Φ,

$$p = \cos\Phi = \frac{\cos\varphi_1 + h_3 k_3 \cos\varphi_3 + \cdots}{(1 + h_3^2 + \cdots)^{\frac{1}{2}}(1 + k_3^2 + \cdots)^{\frac{1}{2}}}, \qquad (1)$$

and Φ may be regarded as the "equivalent phase difference" between the electromotive force and current; Φ is equal to the phase difference between two sine waves having the same effective values as E and I, respectively, and the same mean product of their instantaneous values (see Standardization Rules of the A. I. E. E., Art. 83).

The above equation for cos Φ shows that even when the fundamentals and harmonics of current and electromotive force are, respectively, in the same phase, *i. e.*, φ_1, φ_3, etc., $= 0$, the value of cos Φ cannot be equal to unity; or, the power cannot be expressed

by the product of the effective values of electromotive force and current, unless we have $h_3 = k_3$, $h_5 = k_5$, etc., that is, the wave forms of electromotive force and current are similar to each other.

However, since the cosine assumes the same sign if the sign of the angle be changed, the foregoing equation for cos Φ does not determine the sign of Φ; or, in other words, it is indefinite which of the two alternating quantities is ahead of the other in phase. We may give to Φ a sign equal to that of φ_1, or in case $\varphi_1 = 0$, that of φ corresponding to the lowest harmonic which happens to have a phase difference. But, if all φ's are equal to zero, there is no way of determining the sign of Φ.

On the other hand, the angle Φ is evidently not equal to the angle φ_1, which is the phase difference between the fundamentals of the two waves. In general, also, the angle Φ is not equal to either the phase angle between the zero values of the two waves or the the angle between their maximum values.

The term "equivalent phase difference" has, therefore, no definite physical meaning, and, in case of non-harmonic alternating quantities, is simply a conventional term. Hence, it may happen that the equivalent phase difference between two quantities, as A and C, is neither the sum nor the difference of the phase difference between A and a third quantity, B, and the phase difference between B and C. For example, if B lags 30° behind A, and C lags 30° behind B, C may lag, say, 50° (not 60°) behind A. The angles are related as the three angles of a triangular pyramid.

If the sign of any φ, in the equation for i, is changed, the new equation will represent a current i' of different wave form, but with the same equivalent phase difference with respect to e. The phase difference between i and i' will, however, not be zero. This will be considered more fully later.

Sum, or Difference, of Two Alternating Quantities. — Let the two alternating quantities be expressed by

$$i_1 = I_{1m}\{\sin \omega t + k_{13} \sin (3\omega t + \alpha_3) + \cdots\},$$

and

$$i_2 = I_{2m}\{\sin (\omega t - \varphi_1) + k_{23} \sin (3\omega t + \alpha_3 - \varphi_3) + \cdots\},$$

whose effective values are I_1 and I_2, respectively.

Then the sum, or difference, of the above two alternating quantities is

$$i = i_1 \pm i_2$$
$$= \sqrt{I_{1m}^2 + I_{2m}^2 \pm 2I_{1m}I_{2m}\cos\varphi_1}\,\sin(\omega t - \psi_1)$$
$$+ \sqrt{(k_{13}I_{1m})^2 + (k_{23}I_{2m})^2 + 2k_{13}k_{23}I_{1m}I_{2m}\cos\varphi_3}\,\sin(3\omega t + a_3 - \psi_3)$$
$$+ \cdots,$$

where

$$\psi_1 = \tan^{-1}\frac{\pm I_{2m}\sin\varphi_1}{I_{1m} \pm I_{2m}\cos\varphi_1}, \quad \psi_3 = \tan^{-1}\frac{\pm k_{23}I_{2m}\sin\varphi_3}{k_{13}I_{1m} \pm k_{23}I_{2m}\cos\varphi_3}, \quad \text{etc.}$$

The effective value of i then becomes

$$I = \sqrt{I_1^2 + I_2^2 \pm 2I_1I_2\left\{\frac{\cos\varphi_1 + k_{13}k_{23}\cos\varphi_3 + \cdots}{(1 + k_{13}^2 + \cdots)^{\frac{1}{2}}(1 + k_{23}^2 + \cdots)^{\frac{1}{2}}}\right\}}.$$

But, by reference to equation (1), the fraction within the brackets in this expression is seen to be the cosine of the equivalent phase difference Φ between i_1 and i_2, and we have

$$I = \sqrt{I_1^2 + I_2^2 \pm 2I_1I_2\cos\Phi}. \qquad (2)$$

If Ψ and θ represent the equivalent phase differences between i and i_1, and between i and i_2, respectively, their values can be found from the equations for i_1, i_2, and i; expressed in terms of I_1, I_2, I, and Φ, they are:

$$\Psi = \cos^{-1}\frac{I_1 \pm I_2\cos\Phi}{I}.$$

$$\theta = \cos^{-1}\frac{I_1\cos\Phi \pm I_2}{I}.$$

It is seen that in this case

$$\theta = \Phi \mp \Psi.$$

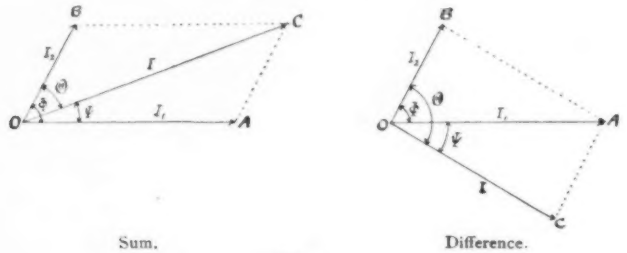
From this result we may conclude that, so far as the sum, or difference, of two alternating quantities are concerned, any non-harmonic alternating quantities can be represented by equivalent simple harmonic alternating quantities which have the same effective values and have a phase difference equal to the equivalent phase difference Φ, between i_1 and i_2, as shown above. Thus, for i_1 and i_2 may be substituted

$$i_1' = \sqrt{2}I_1\sin\omega t,$$

and

$$i_2' = \sqrt{2} I_2 \sin(\omega t - \Phi).$$

Vector Representation of the Sum, or Difference, of Two Alternating Quantities. — According to equation (2) for I already obtained, the relation of I_1, I_2, and I, may be represented by three sides of a



Sum. Difference.

Fig. 1.

triangle. In Fig. 1, if two lines OA and OB are radius-vectors representing I_1 and I_2, then OC is the radius-vector representing their resultant, as is well known in the case of harmonic alternating quantities, and

$$OC = \sqrt{OA^2 + OB^2 \pm 2 \cdot OA \cdot OB \cdot \cos \angle AOB}.$$

The angle between OA and OB is made equal to Φ. It can easily be shown that the angle $AOC = \Psi$, and the angle $BOC = \theta$.

Therefore, by using the conventional term "equivalent phase difference," any two non-harmonic alternating quantities can be represented by radius-vectors, and their addition or subtraction may be performed in the same manner as with simple harmonic alternating quantities. In such cases, therefore, the vector diagram is a plane figure.

From the above result we can also conclude that any alternating quantity can be resolved into two components which have an equivalent phase difference of 90 degrees between them. From the equation (1) for equivalent phase difference, it is also seen that if not only the fundamentals but all the higher harmonics of two alternating quantities have a phase difference of 90 degrees separately, the equivalent phase difference between the two quantities is also equal to 90 degrees.

Sum of Three or More Alternating Quantities. — Let us take any three alternating quantities the instantaneous values of which are expressed by

$$i_1 = I_{1m}\{\sin \omega t + k_{13} \sin (3\omega t + \alpha_3) + \cdots\},$$

$$i_2 = I_{2m}\{\sin (\omega t - \varphi_1) + k_{23} \sin (3\omega t + \alpha_3 - \varphi_3) + \cdots\},$$

and

$$i_3 = I_{3m}\{\sin (\omega t - \psi_1) + k_{33} \sin (3\omega t + \alpha_3 - \psi_3) + \cdots\},$$

their effective values being I_1, I_2, and I_3, respectively. The effective value of their sum is

$$I = \sqrt{I_1^2 + I_2^2 + I_3^2 + 2I_1I_2 \cos \Phi + 2I_1I_3 \cos \Psi + 2I_2I_3 \cos \Theta},$$

where Φ, Ψ, and Θ, denote the equivalent phase differences between i_1 and i_2, between i_1 and i_3, and between i_2 and i_3, respectively. If the equivalent phase differences between i_1 and i, between i_2 and i, and also between i_3 and i, are $(I_1\hat{I})$, $(I_2\hat{I})$, and $(I_3\hat{I})$, respectively, they can be computed from equations for i_1, i_2, and i_3; and their values are as follows:

$$(I_1\hat{I}) = \cos^{-1} \frac{I_1 + I_2 \cos \Phi + I_3 \cos \Psi}{I};$$

$$(I_2\hat{I}) = \cos^{-1} \frac{I_1 \cos \Phi + I_2 + I_3 \cos \Theta}{I};$$

$$(I_3\hat{I}) = \cos^{-1} \frac{I_1 \cos \Psi + I_2 \cos \Theta + I_3}{I}.$$

In a similar manner, any number of alternating quantities can be added together. If I_1, I_2, I_3, etc., are their respective effective values, and $(I_1\hat{I}_2)$, $(I_1\hat{I}_3)$, $(I_2\hat{I}_3)$, etc., are the equivalent phase differences between i_1 and i_2, i_1 and i_3, i_2 and i_3, etc., respectively, the effective value of the resultant will be

$$I = \{I_1^2 + I_2^2 + I_3^2 + \cdots + 2I_1I_2 \cos (I_1\hat{I}_2) + 2I_1I_3 \cos (I_1\hat{I}_3) + \cdots + 2I_2I_3 \cos (I_2\hat{I}_3) + \cdots\}^{\frac{1}{2}}.$$

Vector Representation of Three or More Alternating Quantities. — Take three alternating quantities i_1, i_2, and i_3, and let Φ, Ψ, and Θ, be the equivalent phase differences between each pair of them, as

shown before. If the values of Φ, Ψ, and θ, are computed, it will be found that θ is generally not equal to $(\Phi - \Psi)$ or $(\Phi + \Psi)$. The only relations which can be obtained from equations for them are:

$$\theta \lessdot \Phi - \Psi, \quad \text{and} \quad \theta \gtrdot \Phi + \Psi.$$

Hence, if I_1 and I_2 be represented by radius-vectors OA and OB, respectively, making the angle between them equal to Φ, as in Fig. 2, the third radius-vector OC representing I_3 must be placed outside of the plane containing OA and OB, in order to make the angle AOC equal to Ψ, and the angle BOC equal to θ. In this case, therefore, the vector diagram becomes a space figure, and the diag-

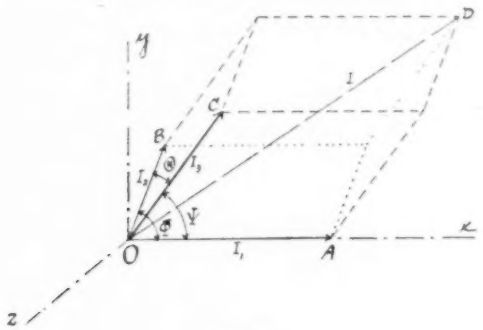


Fig. 2.

onal OD of a parallelopiped with OA, OB, and OC, as its three edges meeting at O, will represent the resultant I.

$$OD = \sqrt{\overline{OA}^2 + \overline{OB}^2 + \overline{OC}^2 + 2 \cdot OA \cdot OB \cos \angle AOB + 2 \cdot OA \cdot OC \cos \angle AOC + 2 \cdot OB \cdot OC \cos \angle BOC},$$

$$= \sqrt{I_1^2 + I_2^2 + I_3^2 + 2I_1I_2 \cos \Phi + 2I_1I_3 \cos \Psi + 2I_2I_3 \cos \theta}.$$

It can also be shown that equivalent phase differences $(I_1\hat{I})$, $(I_2\hat{I})$, and $(I_3\hat{I})$, are represented by angles AOD, BOD, and COD, respectively. It is seen, then, that any three alternating quantities can be represented by radius-vectors arranged in space, so that the vector diagram becomes a space figure.

Four or more alternating quantities can be treated in a similar manner, but in this case an idea of a vector diagram in four or more dimensions must be introduced in order to represent them simul-

taneously by radius-vectors. The vector diagram, therefore, can no longer be expressed by any geometrical figure. In general, if there are n independent alternating quantities to be represented simultaneously by radius-vectors, the vector diagram must become a figure of n dimensions.

However, in some special cases radius-vectors representing n independent alternating quantities can be arranged in a figure of less than n dimensions. Taking the case of three alternating quantities mentioned before, if θ is equal to either $(\Phi - \Psi)$ or $(\Phi + \Psi)$, the radius-vector OC will fall in the plane AOB, so that the vector diagram becomes a figure of two dimensions or a plane figure. Let us see what are the conditions for such special cases.

Let the three alternating quantities be i_1, i_2, and i_3, as shown before, and suppose i_2 and i_3 are so resolved into i_2' and i_2'', and i_3' and i_3'', respectively, that i_2' and i_3' have the same phase and wave form as i_1, while i_2'' and i_3'' are their remaining parts of which the fundamentals are in quadrature with i_1. Then

$$i_2' = I_{2m} \cos \varphi_1 \{\sin \omega t + k_{13} \sin (3\omega t + a_3) + \cdots\};$$

$$i_2'' = I_{2m} \{\sin \varphi_1 \cos \omega t + k_{23} \sin \varphi_3 \cos (3\omega t + a_3) + \cdots + (k_{23} \cos \varphi_3 - k_{13} \cos \varphi_1) \sin (3\omega t + a_3) + \cdots\};$$

$$i_3' = I_{3m} \cos \psi_1 \{\sin \omega t + k_{13} \sin (3\omega t + a_3) + \cdots\};$$

$$i_3'' = I_{3m} \{\sin \psi_1 \cos \omega t + k_{33} \sin \psi_3 \cos (3\omega t + a_3) + \cdots + (k_{33} \cos \psi_3 - k_{13} \cos \psi_1) \sin (3\omega t + a_3) + \cdots\}.$$

The resultant of i_1, i_2, and i_3, is evidently equal to the resultant of i_1, i_2', i_2'', i_3', and i_3''. But i_1, i_2', and i_3', are in phase and have similar wave forms, and therefore the equivalent phase difference between any pair of them is zero, so that the radius-vector representing the sum of these three is simply the algebraic sum of the radius-vectors representing them separately. And if i' is the sum of these three components i_1, i_2', and i_3', and if I_1, I_2', I_3', and I', are radius-vectors representing i_1, i_2', i_3', and i', respectively, then

$$I' = I_1 + I_2' + I_3'.$$

In case i_2'' and i_3'' have similar wave forms, the radius-vector I'' representing the sum of the two is equal to the algebraic sum of the radius-vectors I_2'' and I_3'' representing i_2'' and i_3'', respectively;

$$I'' = I_2'' + I_3''.$$

Then in this case, since the radius-vectors I_1, I_2', and I_3', are all in one direction, and I_2'' and I_3'' are in another direction, the radius-vectors I_1, I_2 (resultant of I_2' and I_2''), and I_3 (resultant of I_3' and I_3''), which represent i_1, i_2, and i_3, respectively, must lie in one plane, or the diagram must be a plane figure.

Now, the conditions that i_2'' and i_3'' have similar wave forms are, for the third harmonic,

$$k_{23}\frac{\sin\varphi_3}{\sin\varphi_1} = k_{33}\frac{\sin\psi_3}{\sin\psi_1},$$

and

$$\frac{k_{23}\cos\varphi_3 - k_{13}\cos\varphi_1}{\sin\varphi_1} = \frac{k_{33}\cos\psi_3 - k_{13}\cos\psi_1}{\sin\psi_1},$$

with similar conditions for other harmonics. If the conditions shown above are satisfied by k's, φ's, and ψ's, then the vector diagram for the three alternating quantities will become a plane figure. The above consideration may be extended to any number of alternating quantities.

These conditions are found in a series circuit with resistance and inductance or resistance and capacity (but not resistance, inductance and capacity) and in the arrangements of circuits known as the "three-voltmeter" method and "three-ammeter" method discussed later. So far the writer has found no other cases.

Graphical Method of Addition and Subtraction of Non-harmonic Alternating Quantities. — We have seen that when three or more alternating quantities are to be represented by radius-vectors simultaneously, an idea of a vector diagram in three or more dimensions is necessary. But, even in such cases, when any two of them are taken separately, they can be represented by radius-vectors in a plane figure. Hence, summation and subtraction of any number of alternating quantities may be performed graphically, representing them by radius-vectors, by first taking any two and finding the resultant by a plane vector diagram, combining the resultant with a

third in a separate plane diagram, combining the new resultant with a fourth, etc.

Let us take three alternating quantities i_1, i_2, and i_3, whose effective values are I_1, I_2, and I_3, respectively, as shown before. Then the effective value I' of the sum of i_1 and i_2 is

$$I' = \sqrt{I_1^2 + I_2^2 + 2I_1 I_2 \cos (\hat{I_1 I_2})},$$

where $(\hat{I_1 I_2})$ represents the equivalent phase difference between i_1 and i_2.

Now, if $(\hat{I' I_3})$ is the equivalent phase difference between I' and I_3, its value is given by

$$\cos (\hat{I' I_3}) = \frac{I_1 \cos (\hat{I_1 I_3}) + I_2 \cos (\hat{I_2 I_3})}{I'}, \qquad (3)$$

where $(\hat{I_1 I_3})$ and $(\hat{I_2 I_3})$ are the equivalent phase differences between I_1 and I_3, and between I_2 and I_3, respectively.

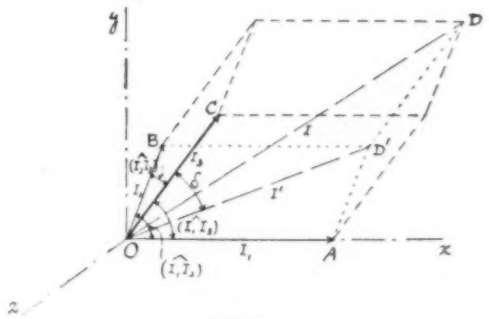


Fig. 3.

On the other hand, if i_1, i_2, and i_3, are represented by radius-vectors OA, OB, and OC, respectively (Fig. 3), the resultant of i_1 and i_2, that is i', will be represented by the radius-vector OD' which is the diagonal of a parallelogram with OA and OB at its two sides. If the angle between OD' and OC is denoted by δ, then we must have

$$OD' \cos \delta = OA \cdot \cos \angle AOC + OB \cdot \cos \angle BOC.$$

Hence

$$\cos \delta = \frac{OA \cdot \cos \angle AOC + OB \cos \angle BOC}{OD'}$$

$$= \frac{I_1 \cos (\hat{I_1 I_3}) + I_2 \cos (\hat{I_2 I_3})}{I'}.$$

Comparing this result with the equation for $\cos(I'\hat{I}_3)$ obtained before, it is seen that the angle δ represents the equivalent phase difference between i' and i_3.

A similar consideration can be extended to the case in which any number of alternating quantities are treated. Suppose, for example, we have another alternating quantity i_4 whose effective value is I_4; then the equivalent phase difference between I' and I_4 is given by

$$\cos(I'\hat{I}_4) = \frac{I_1 \cos(I_1\hat{I}_4) + I_2 \cos(I_2\hat{I}_4)}{I'}.$$

If i'' is the resultant of i' and i_3, its effective value I'' is

$$I'' = \sqrt{I'^2 + I_3^2 + 2I'I_3 \cos(I'\hat{I}_3)},$$

and the equivalent phase difference between I'' and I_4 is given by

$$\cos(I''\hat{I}_4) = \frac{I' \cos(I'\hat{I}_4) + I_3 \cos(I_3\hat{I}_4)}{I''},$$

$$= \frac{I_1 \cos(I_1\hat{I}_4) + I_2 \cos(I_2\hat{I}_4) + I_3 \cos(I_3\hat{I}_4)}{I''}.$$

From the above results we can derive a method of adding or subtracting any number of alternating quantities by means of separate vector diagrams, each of which is a plane figure. For instance, suppose we have four alternating quantities represented by A, B, C, and D, and the equivalent phase differences between each pair of them are all known to be θ_{AB}, θ_{AC}, θ_{AD}, θ_{BC}, θ_{BD}, and θ_{CD}. Let M be the resultant of A and B; then M is found as a diagonal of a parallelogram whose two adjacent sides are A and B, the diagram being a plane figure, as shown in Fig. 4. If θ_{MC} is the equivalent phase

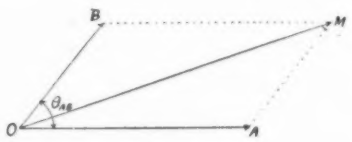


Fig. 4.

difference between M and C, we have, according to equation (3),

$$\cos\theta_{MC} = \frac{A\cos\theta_{AC} + B\cos\theta_{BC}}{M}.$$

The magnitude of θ_{MC} may be found graphically as shown in Fig. 5. Draw OM equal in length to OM in Fig. 4. Draw OA equal in length to OA in Fig. 4, making an angle $MOA = \theta_{AC}$. Drop a perpendicular AR from A upon OM. From R draw a line RB equal in length to OB in Fig. 4, making an angle $MRB = \theta_{BC}$; and drop a perpendicular BS from B upon OM. Strike a semicircle upon OM, and take a point K upon it so that $OK = OS$. Then the angle $KOM = \theta_{MC}$. For

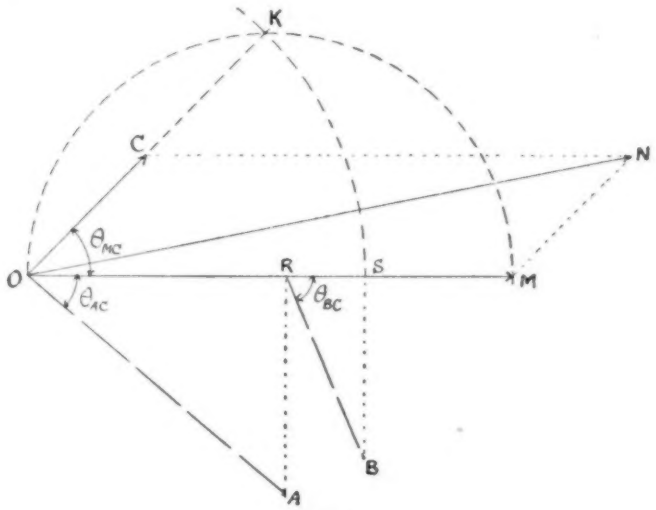


Fig. 5.

$$\cos \angle KOM = \frac{OK}{OM} = \frac{OS}{OM} = \frac{OA \cdot \cos \angle AOR + RB \cdot \cos \angle BRS}{OM},$$

$$= \frac{A \cos \theta_{AC} + B \cos \theta_{BC}}{M} = \cos \theta_{MC};$$

therefore,

$$\angle KOM = \theta_{MC}.$$

Thus θ_{MC} having been known, the resultant N of M and C is found to be represented by the radius-vector ON which is the diagonal of a parallelogram with OC and OM as its two sides, as in Fig. 5.

If in the above procedure, the point S happens to fall on the portion of the line MO, produced the other side of O, the point K should be taken on the semicircle drawn on the lower side of OM,

and OC must be put on the produced portion of KO, so that the angle θ_{MC} will become greater than 90°.

If θ_{ND} is the equivalent phase difference between the resultant N and the fourth quantity D, its magnitude may be found graphically in a similar manner, as shown in Fig. 6. In this case the point T,

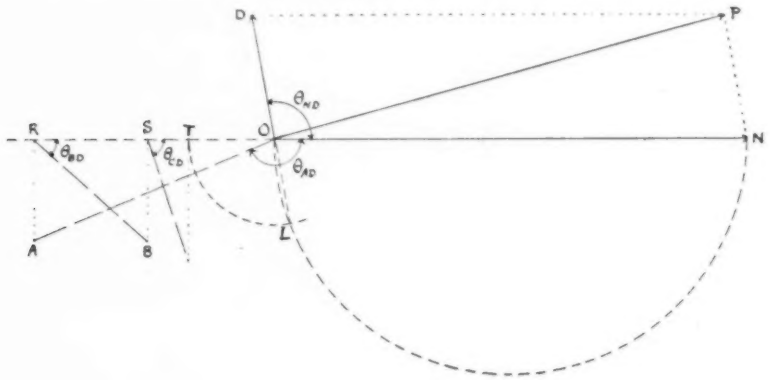


Fig. 6.

the foot of the last perpendicular, happens to fall outside of NO, so that the point L is taken on the lower semicircle, and OD is put on the produced portion of the line LO, as seen in the figure. It is also easily shown that the angle DON thus obtained is equal to θ_{ND}. Thus θ_{ND} having been known the next step is only to construct the parallelogram $ODPN$ whose two sides ON and OD represent the resultant N and the fourth quantity D, respectively. Then the diagonal OP will be the radius-vector representing the resultant of N and D, which is therefore the resultant of the given four alternating quantities A, B, C and D.

If some of these quantities are to be subtracted, the radius-vectors representing them should be drawn in the opposite direction in each diagram; thus, for instance, if the third quantity C is to be subtracted, Fig. 5 and Fig. 6 will be changed to Fig. 7 and Fig. 8, respectively.

Any number of radius-vectors representing non-harmonic alternating quantities can be treated in a similar manner.

Non-harmonic Alternating Electromotive Force and Current in a Circuit. — If a non-harmonic alternating electromotive force is

applied to a circuit the wave form of current will generally be different from that of the electromotive force ; the only case in which these two waves can have similar forms is when the circuit contains nothing but non-inductive resistances.

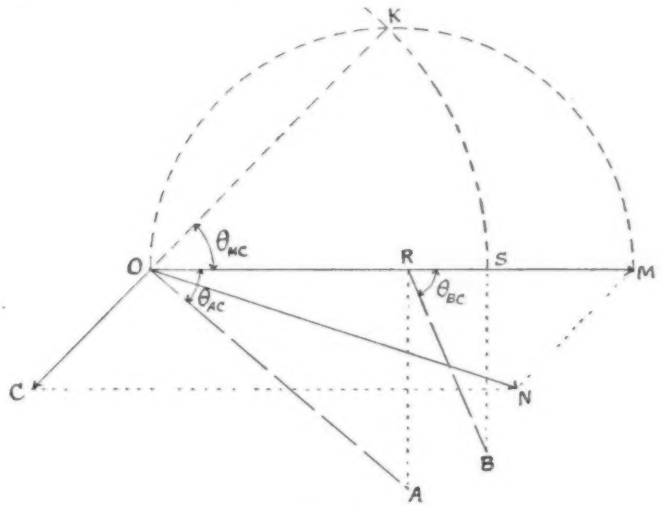


Fig. 7.

If the circuit contains inductance the percentage amplitude of any harmonic in the current wave is always less than that of the

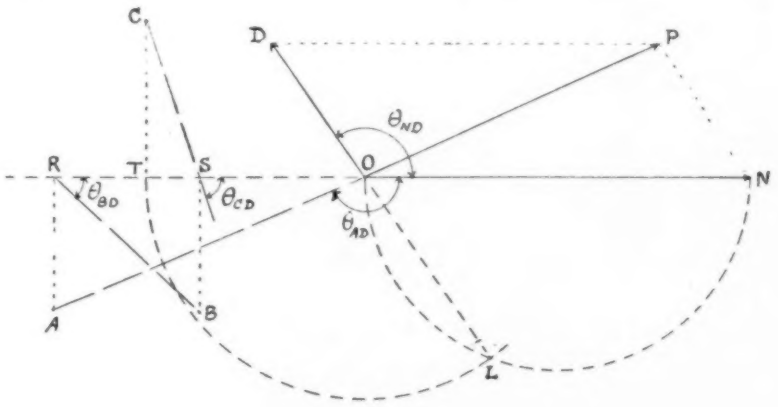


Fig. 8.

corresponding harmonic in the electromotive force wave. If the circuit contains capacity but no inductance, any harmonic in the

current wave has a greater percentage amplitude than the corresponding harmonic in the electromotive force wave.

If the circuit contains both inductance and capacity connected in series, the percentage amplitude of a harmonic in the current wave may be less or greater than that of the corresponding harmonic in the electromotive force wave. If L and C are the inductance and the capacity, respectively, and ω is equal to 2π times the frequency of the fundamental wave, then the harmonics of lower order than $(1/\omega^2 LC)$th have greater percentage amplitudes in the current than in the electromotive force wave, but with those of higher order than $(1/\omega^2 LC)$th, the percentage amplitudes are less in the current than in the electromotive force wave.

If the current in the circuit is expressed by

$$i = I_m\{\sin \omega t + k_3 \sin(3\omega t + a_3) + k_5 \sin(5\omega t + a_5) + \cdots\}$$

whose effective value is I, the effective value of the electromotive force is:

(1) for a circuit containing resistance R and inductance L in series,

$$E = I\sqrt{R^2 + (\omega L\rho)^2};$$

(2) for a circuit containing resistance R and capacity C in series,

$$E = I\sqrt{R^2 + \frac{1}{(\omega C\lambda)^2}};$$

(3) for a circuit containing resistance R, inductance L, and capacity C, in series,

$$E = I\sqrt{R^2 + (\omega L\rho)^2 + \frac{1}{(\omega C\lambda)^2} - \frac{2L}{C}},$$

where ρ and λ are

$$\rho = \sqrt{\frac{1 + (3k_3)^2 + (5k_5)^2 + \cdots}{1 + k_3^2 + k_5^2 + \cdots}},$$

$$\lambda = \sqrt{\frac{1 + k_3^2 + k_5^2 + \cdots}{1 + \left(\frac{k_3}{3}\right)^2 + \left(\frac{k_5}{5}\right)^2 + \cdots}},$$

which may be called "correction factors" for inductance and capacity, respectively.

The equivalent phase difference Φ between the electromotive force and the current is :

(1) for a circuit containing R and L in series,

$$\Phi = \cos^{-1} \frac{R}{\sqrt{R^2 + (\omega L \rho)^2}};$$

(2) for a circuit containing R and C in series,

$$\Phi = \cos^{-1} \frac{R}{\sqrt{R^2 + \frac{1}{(\omega C \lambda)^2}}};$$

(3) for a circuit containing R, L, and C, in series,

$$\Phi = \cos^{-1} \frac{R}{\sqrt{R^2 + (\omega L \rho)^2 + \frac{1}{(\omega C \lambda)^2} - \frac{2L}{C}}},$$

where, in each case, cos Φ is equal to the power-factor of the circuit.

The equivalent phase difference between the current and the resistance voltage IR is zero in each case ; and the equivalent phase difference between the current and the inductance voltage $I(\omega L \rho)$, and also that between the current and the capacity voltage $I/(\omega C \lambda)$, are equal to 90 degrees in each case ; but the equivalent phase difference between the inductance voltage and the capacity voltage is equal to $\cos^{-1}(-\lambda/\rho)$, which is not equal to 180 degrees. The vector diagram showing the relation of the resistance voltage, the inductance voltage, and the capacity voltage of a circuit must, therefore, become a space figure. When, of the three kinds of quantities, resistances, inductances and capacities, only two kinds are present in a series circuit, the vector diagram becomes a plane figure. In ordinary cases, however, ρ and λ are nearly equal to unity, so that $\cos^{-1}(-\lambda/\rho)$ is nearly equal to 180 degrees.

Verification of Three-voltmeter Method and Three-ammeter Method of Power Measurement.— As an example of the vector representation of non-harmonic alternating quantities, we shall take the case of the so-called three-voltmeter method and three-ammeter method of power measurement, and show that these methods of power measurements are true for any nature of circuit and for alternating cur-

rents of any wave form. In both of these methods of power measurement, a certain known non-inductive resistance R is used in connection with the load circuit, the power of which is to be measured.

(a) Three-voltmeter method.

In this method, a resistance R is connected in series with the load circuit; if E_1, E_2, and E, are the effective values of the potential differences across the load circuit, across the resistance R, and across the whole circuit, respectively, the formula for the power in the load circuit is

$$W = \frac{(E^2 - E_1^2 - E_2^2)}{2R}.$$

Let I be the effective value of current in the circuit, and Φ be the equivalent phase difference between I and E_1. Since E_2 is the voltage across the non-inductive resistance R, its wave form is similar to that of I, so that the equivalent phase difference between I and E_2 is zero. Therefore, although we have three independent alternating quantities, viz., E_1, E_2, and I, yet they can be represented by radius-vectors in a plane vector diagram.

Since the power-factor of the load circuit is equal to cos Φ, the power in that circuit is

$$W = E_1 I \cos \Phi.$$

But since

$$I = \frac{E_2}{R},$$

and it is seen from the vector diagram that

$$\cos \Phi = \frac{E^2 - E_1^2 - E_2^2}{2E_1E_2},$$

we have

$$W = \frac{E^2 - E_1^2 - E_2^2}{2R}.$$

Thus, this method of power measurement can be used for any circuit without any restriction, whatever the nature of the load circuit and whatever the wave forms of E_1 and I.

(b) Three-ammeter method.

In this method, a resistance R is connected in parallel with the load circuit; if I_1, I_2, and I, are the effective values of currents in

the load circuit, in the resistance R, and in the main line, respectively, the formula for the power in the load circuit is

$$W = \tfrac{1}{2}R(I^2 - I_1^2 - I_2^2).$$

Let E be the effective value of the electromotive force applied to the circuit, and Φ be the equivalent phase difference between E and I_1. Then, by the same reasoning as in the previous case, the radius-vectors representing I_1, I_2, I, and E, are in a plane diagram.

The power in the load circuit is again expressed by

$$W = EI_1 \cos \Phi.$$

But we have $E = I_2R$, and from the diagram we have

$$\cos \Phi = \frac{I^2 - I_1^2 - I_2^2}{2I_1I_2};$$

therefore

$$W = \tfrac{1}{2}R(I^2 - I_1^2 - I_2^2).$$

Thus, the three-ammeter method of power measurement can also be used for any circuit, whatever the nature of the load circuit, and whatever the wave forms of electromotive force and current.

Power-factor and Reactive-factor. The power-factor of a circuit is defined as "the ratio of the electric power in watts to the apparent power in volt-amperes,"[1] or if p is the power-factor, $p =$ watts $\div$ volt-amperes.

If both electromotive force and current waves are sinusoidal, the power-factor can also be expressed by either energy voltage $\div$ total voltage or energy current $\div$ total current, and is equal to the cosine of the phase difference between the electromotive force and the current.

In the case of non-harmonic alternating currents, however, the power is equal to the product of the effective values of the electromotive force and the current, multiplied by the cosine of the equivalent phase difference Φ between them. Thus the power-factor p is equal to cos Φ, as in equation (1).

Let the electromotive force and the current be

$$e = E_m\{\sin \omega t + h_3 \sin (3\omega t + \theta_3) + \cdots\},$$

and

$$i = I_m\{\sin (\omega t - \varphi_1) + k_3 \sin (3\omega t + \theta_3 - \varphi_3) + \cdots\},$$

[1] Standardization Rules, A. I. E. E., Art. 54.

whose effective values are E and I, respectively; and suppose the current i to be resolved into two components, an energy current i_w and a wattless current i_l. The energy current is that part of the current i which has its fundamental and all higher harmonics in the same phase as the fundamental and corresponding harmonics of the electromotive force, and the wattless component is the remaining part of the current i, and has its fundamental and all higher harmonics in quadrature with the fundamental and harmonics, respectively, of the electromotive force wave. Therefore, these two components can be expressed by

$$i_w = I_m \cos \varphi_1 \left\{ \sin \omega t + k_3 \frac{\cos \varphi_3}{\cos \varphi_1} \sin (3\omega t + \theta_3) + \cdots \right\},$$

and

$$i_l = - I_m \sin \varphi_1 \left\{ \cos \omega t + k_3 \frac{\sin \varphi_3}{\sin \varphi_1} \cos (3\omega t + \theta_3) + \cdots \right\},$$

whose effective values are I_w and I_l, respectively.

The equivalent phase difference between the electromotive force e and the wattless current i_l is 90 degrees, but that between e and the energy current i_w is generally not equal to zero, being equal to Φ' and is given by

$$\cos \Phi' = \frac{\cos \varphi_1 + h_3 k_3 \cos \varphi_3 + \cdots}{(1 + h_3^2 + \cdots)^{\frac{1}{2}} (\cos^2 \varphi_1 + k_3^2 \cos^2 \varphi_3 + \cdots)^{\frac{1}{2}}}.$$

Similarly, if the electromotive force e is resolved into an energy voltage e_w, which has its fundamental and all higher harmonics in the same phase as the fundamental and corresponding harmonics in the current, and a wattless voltage e_l, which has its fundamental and all higher harmonics in quadrature to the fundamental and harmonics, respectively, in the current wave, these component voltages are

$$e_w = E_m \cos \varphi_1 \left\{ \sin (\omega t - \varphi_1) + h_3 \frac{\cos \varphi_3}{\cos \varphi_1} \sin (3\omega t + \theta_3 - \varphi_3) + \cdots \right\},$$

and

$$e_l = E_m \sin \varphi_1 \left\{ \cos (\omega t - \varphi_1) + h_3 \frac{\sin \varphi_3}{\sin \varphi_1} \cos (3\omega t + \theta_3 - \varphi_3) + \cdots \right\},$$

whose effective values are E_w and E_l, respectively.

The equivalent phase difference between the current i and the wattless voltage e_i is 90 degrees; the phase difference between i and the energy voltage e_w is, however, generally not equal to zero, but is Φ'', which is given by

$$\cos \Phi'' = \frac{\cos \varphi_1 + h_3 k_3 \cos \varphi_3 + \cdots}{(\cos^2 \varphi_1 + h_3^2 \cos^2 \varphi_3 + \cdots)^{\frac{1}{2}}(1 + k_3^2 + \cdots)^{\frac{1}{2}}}.$$

The power W may then be expressed by either $EI_w \cos \Phi'$ or $E_w I \cos \Phi''$; hence

$$W = EIp = EI_w \cos \Phi' = E_w I \cos \Phi'',$$

therefore

$$p = \frac{I_w}{I} \cos \Phi' = \frac{E_w}{E} \cos \Phi''.$$

For circuits carrying non-harmonic alternating currents, therefore, the power-factor is generally equal to neither $\frac{\text{energy current}}{\text{total current}}$ nor $\frac{\text{energy voltage}}{\text{total current}}$, except in special cases, being equal to the former when $\Phi' = 0$ and equal to the latter when $\Phi'' = 0$.

The reactive-factor of a circuit is defined as "the ratio of the wattless volt-amperes (*i. e.*, the product of the wattless component of current by voltage, or wattless component of voltage by current) to the total volt-amperes."[1] If the electromotive force and the current are of sinusoidal wave form the wattless volt-amperes are equal to the total volt-amperes multiplied by the sine of the phase difference φ between the electromotive force and the current, so that the reactive-factor q is equal to $\sin \varphi$, and also be expressed by either $\frac{\text{wattless current}}{\text{total current}}$ or $\frac{\text{wattless voltage}}{\text{total voltage}}$.

But when the electromotive force and the current are non-harmonic the product of the wattless component of current by voltage, $I_i E$, is generally not equal to the product of the wattless component of voltage by current, IE_i. Hence the above definition given to the term "wattless volt-amperes" does not show completely what it is. The writer ventures to suggest the following definition for this term, or "wattless power" as it is sometimes called:

[1] Standardization Rules, A. I. E. E., Art. 55.

The "wattless power" in a circuit is the power which would be given by the actual electromotive force in the circuit and a fictitious current having a fundamental and higher harmonics that are equal in amplitude to the fundamental and corresponding harmonics of the actual current but are in quadrature thereto, respectively, in phase; or, by the actual current in the circuit and a fictitious electromotive force having a fundamental and higher harmonics that are equal in amplitude to the fundamental and corresponding harmonics of the actual electromotive force but are in quadrature thereto, respectively, in phase.

If e and i are the electromotive force and the current in a circuit, as given above, the wattless power is the power given either by the electromotive force e and the fictitious current i', where

$$i' = I_m\{\cos(\omega t - \varphi_1) + k_3 \cos(3\omega t + \theta_3 - \varphi_3) + \cdots\},$$

or by the current i and the fictitious electromotive force e', where

$$e' = E_m\{\cos \omega t + h_3 \cos(3\omega t + \theta_3) + \cdots\}.$$

The equivalent phase difference between e and i' is equal to that between e' and i, and will be expressed by

$$\Psi = \cos^{-1} \frac{\sin \varphi_1 + h_3 k_3 \sin \varphi_3 + \cdots}{(1 + h_3^2 + \cdots)^{\frac{1}{2}}(1 + k_3^2 + \cdots)^{\frac{1}{2}}}. \qquad (4)$$

Therefore, the wattless power W_l is

$$W_l = IE \cos \Psi,$$

and the reactive-factor q is

$$q = \cos \Psi.$$

The expression for reactive-factor might have been obtained by writing $\sin \varphi_1$, $\sin \varphi_3$, etc., for $\cos \varphi_1$, $\cos \varphi_3$, etc., in equation (1) for power-factor.

The wattless power W_l is generally equal to neither $I_l E$ nor $I E_l$, and consequently the reactive-factor q is generally equal to neither

$$\frac{I_l}{I} = \frac{\text{wattless current}}{\text{total current}},$$

nor

$$\frac{E_l}{E} = \frac{\text{wattless voltage}}{\text{total voltage}}.$$

The value of $p^2 + q^2$ is always less than unity for non-harmonic alternating currents, except when the circuit has no reactance, in which case $p = 1$ and $q = 0$; but it becomes equal to unity for sinusoidal currents.

Conclusions.

1. The relative phase positions of two non-harmonic alternating quantities can be expressed by the term "equivalent phase difference," which is equal to the phase difference between two equivalent sinusoidal alternating quantities that give the same value of mean product of instantaneous values as the given non-harmonic alternating quantities.

2. Any non-harmonic alternating quantities can be represented by radius-vectors, if their relative phase positions are expressed by equivalent phase differences between each pair.

3. Generally, a vector diagram in n dimensions is required to show n independent non-harmonic alternating quantities simultaneously; but in special cases they can be shown in a figure of less dimensions.

4. Summation or subtraction of any number of non-harmonic alternating quantities can be performed graphically, by representing them by radius-vectors and combining first two of them in a plane diagram, combining their resultant with a third, etc.

5. The power-factor, or the ratio of the electric power in watts to the apparent power in volt-amperes in a circuit carrying non-harmonic alternating currents, cannot be expressed by the ratio of energy current to total current, nor by the ratio of energy voltage to total voltage, except in some special cases.

The reactive-factor of a circuit is the ratio of the "wattless power" in watts (as already defined) to the apparent power in volt-amperes. With simple harmonic alternating currents it can be expressed by the ratio of wattless current to total current, or by the ratio of wattless voltage to total voltage; but with non-harmonic alternating currents it can be expressed by neither of them, except in some special cases.

The sum of squares of power-factor and reactive-factor of a circuit is always less than unity with non-harmonic alternating currents. With simple harmonic alternating currents, it is equal to 1.

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A THERMO-HYSTERETIC FREQUENCY METER AND THE APPLICATION OF DIFFERENTIAL THERMO-JUNCTIONS TO A.C.-D.C. COMPARISON.

BY C. E. HIATT.

THE great necessity of knowing the frequency of alternating and oscillating currents has resulted in the invention of numerous devices for frequency determination, but very few, if any, have proven wholly satisfactory. A careful examination of the scientific and technical literature has resulted in finding that only one attempt has been made to utilize as a measure of frequency the rate of heat production when cyclic reversals of magnetic flux are produced in iron, say, by an alternating magnetic field. Meyer [1] investigated the elongation of a steel wire stretched in the axis of a solenoid carrying an alternating current. Careful shielding was necessary and even then the calibration was difficult on account of eddy currents and Joule heat from the coil. The power consumption was also quite large. Guye and Herzfeld [2] might have applied the bolometer method they were using in the study of kinetic hysteresis to the determination of frequency, but they were apparently considering only the problem at hand.

It is the object of this paper to describe first an instrument which is free from the defects mentioned above, but which depends for its action on the rate at which heat is generated by the cyclic reversals of magnetic flux in a few milligrams of iron, say, when placed in a solenoid carrying the current whose frequency is desired. The necessity of empirical calibration has, however, not been eliminated.

A fine iron wire a few millimeters in length was soldered to a copper constantan junction electrically opposed to a similar one thermally and electrically equivalent but not containing iron. This is shown in the heavy line detail drawing of Fig. 1. The junction

[1] Meyer, Elec. Rev., London, Feb. 19, 1897.
[2] Guye and Herzfeld, C. R., 136, p. 957, 1903.

J_2 contains iron while J_1 does not. So far this equivalence has been obtained only after repeated trials, after which both junctions were placed in a solenoid carrying a constant alternating current. A variation in the frequency of this current would cause a proportional variation in the temperature of the junction containing the iron, thus producing a thermo E.M.F. readily observed by a galvanometer. The use of the differential thermo-junction does away with the necessity of excessive shielding and eliminates the troubles

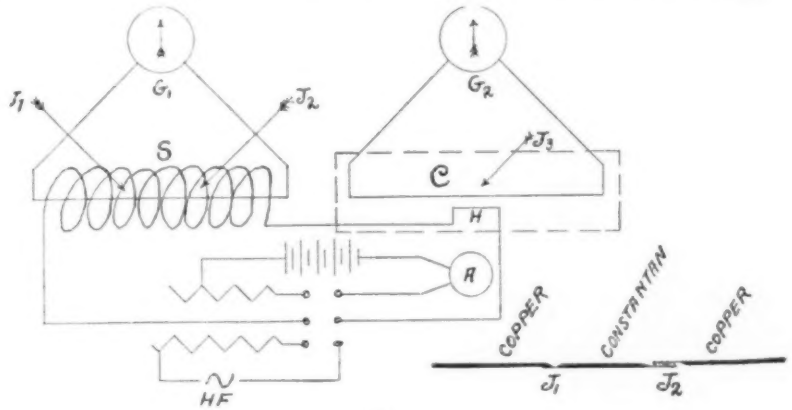

Fig. 1.

which might arise from Joule heat generated in the coil. The slight disturbing effect which may be present as the result of eddy currents will be considered later.

Rapidity of action is secured by making the heat capacity of the junctions small and their radiating power high. This, fortunately, is compatible with the practical requirement that the energy consumption of the measuring instrument be small. So small, in fact, is the energy necessary for the magnetization and demagnetization of such minute bits of metal that it seems wholly practical to use several differential thermo-junctions in series, thus permitting the use of a less sensitive galvanometer. For the sake of simplicity and because the galvanometer available for their investigation was sufficiently sensitive, only one set of differential junctions has been used at a time.

The arrangement of the apparatus is shown in Fig. 1. J_1 and J_2 are the opposed copper constantan junctions, J_2 containing the iron.

The wires used were no. 27 B. & S. gauge constantan and no. 32 B. & S. gauge copper, and were connected to a Leeds and Northrup galvanometer G_1, of the type "H.S.," resistance 129 ohms, sensibility 446 megohms. The length of the constantan between J_1 and J_2 was about 2.5 cm. The solenoid S was 4.4 cm. long and consisted of 662 turns of no. 24 B. & S. gauge copper wire having a resistance of 3.6 ohms. The alternating current through the solenoid was kept at a constant and known value by means of the regulating resistance and frequent comparison with a standard direct current as measured by the Weston ammeter, A. This was accomplished with the help of the comparator C, shown enclosed by the hatched line rectangle in Fig. 1. The comparator consisted of a strip of bismuth about 3.5 cm. long soldered end on to a similar one of antimony, forming a junction J_3 just over the heater H. The strips for 5 or 6 mm. on each side of the junction were made thin in order to keep down the heat capacity and permit speed in working. The heater was of manganin with a resistance of 1.8 ohms and extremely small self-inductance and capacity. The heater and junction were mounted on an insulator and wholly enclosed by a cast-iron dome weighing 40 pounds or more supported by a cast-iron plate over an inch in thickness. This arrangement shielded the instrument perfectly, in fact, much better than was necessary. S was also shielded to make sure that no unequal heating could exist from any cause other than radiation from the solenoid. In order to insure a thoroughly steady condition about the junctions, a direct current equal to the alternating current to be used in any determination was passed through the solenoid for some time before taking observations. This was found unnecessary with the balanced junctions and was generally done to examine the permanency of the balancing. The heater wire H was always in series with the solenoid as shown in the figure. Two wires for this purpose and the two from the junction J_3 to the galvanometer G_2 (also Leeds and Northrup, type "P," resistance 121 ohms, sensibility 106 megohms) were run through a cork in the top of the dome and sealed in with "universal wax." The performance of this simple A.C. instrument was most satisfactory as regards constancy and sensibility, but it possessed to a moderate degree the defect of all hot-wire instruments

of requiring a few seconds to reach its final deflection. It readily permitted all currents used to be kept constant to within one tenth of one per cent.

The high frequency alternator, H.F. was of the inductor type with 16 teeth in the rotating disc and 5 teeth of equal cross-section on each of the poles at opposite ends of a diameter. The teeth were arranged in the arc of a circle concentric with the rotating disc. This disc was of cast iron, 5.5 in. in diameter and .5 in. thick. Between it and the stationary laminated pole pieces there was about .35 in. of clearance. Around each polar tooth there were fifty turns of no. 28 B. & S. gauge double silk-covered copper wire all connected in series. The disc was rotated by means of a pulley and belt connection to a three fourths horse-power motor run by storage cells. Two storage cells were used on the field. The inductor was, in general, similar to one made by Duddell,[1] but possessed the additional feature that the numerous pole pieces kept the reluctance of the magnetic circuit practically constant, a feature which cannot exist when "two chisel-shaped magnet poles" are placed on either side of a rotating notched disc. For suggesting this type of machine and for assistance in its design and construction I am indebted to Dr. R. H. Hough.

To find the particular grade of iron which would give the most heat per hysteretic cycle a number of similar test samples of equal weights were made up. Quite a collection of steel and iron sheets and wires having a wide range of chemical and physical properties were available. Two different samples would be selected and soldered to each of two equal and opposed junctions. When placed in the same alternating field as J_1 and J_2 in Fig. 1, the direction of the galvanometer deflection gave positive evidence as to which became the warmer. Of course it was necessary to have the pair of opposed junctions well balanced at the beginning. The reliability of this balancing could be tested by sending sufficient direct current through the solenoid to make it actually hot.

The speed of the alternator was measured with a Starrett speed indicator and stop watch, the shaft of the indicator being positively

[1] Duddell, Proceedings of the Physical Society of London, April, 1905, Vol. XIX., p. 431.

but flexibly connected to the alternator shaft by a short piece of high grade rubber tubing. The number of revolutions per second of the inductor's disc multiplied by the number of teeth it contains gives the frequency of the alternating current through the solenoid. In the following tables column F denotes this frequency, D corresponding deflections in centimeters of G_1 with the scale at .5 m. distance. Tables I., II. and III. are for 0.15, 0.10 and 0.05 amperes respectively, using a piece of the best iron obtainable, 0.8 cm. long, 0.02 cm. thick and 0.08 cm. wide.

I.		II.		III.	
F	D	F	D	F	D
400	1.70	333	0.70	440	0.40
471	2.20	400	0.93	666	0.61
513	2.61	500	1.26	770	0.70
615	3.11	588	1.55	909	0.80
727	4.06	678	1.80	1,066	0.98
808	4.80	833	2.40	1,230	1.12
899	5.69	941	2.91	1,290	1.20
1,066	7.63	1,116	3.97	1,350	1.31
1,290	9.85	1,230	4.40		
1,404	11.21	1,333	4.91		

These data are plotted in curves I., II. and III., Fig. 2.

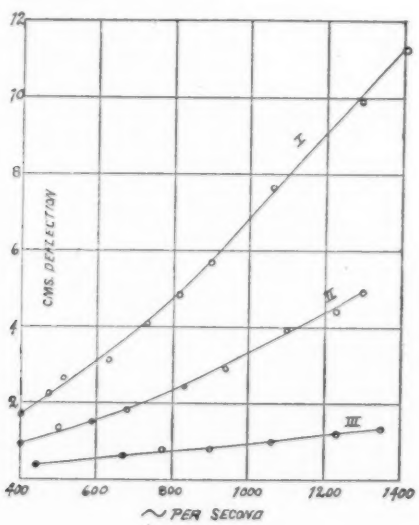

Fig. 2.

The data for Tables IV. and V. were obtained with a piece of tempered Swedish steel of the highest grade and of the same size as the iron specimen used for Tables I., II. and III. These tables are for 0.10 and 0.05 amperes respectively. To facilitate comparison curve II. for iron is repeated in Fig. 3.

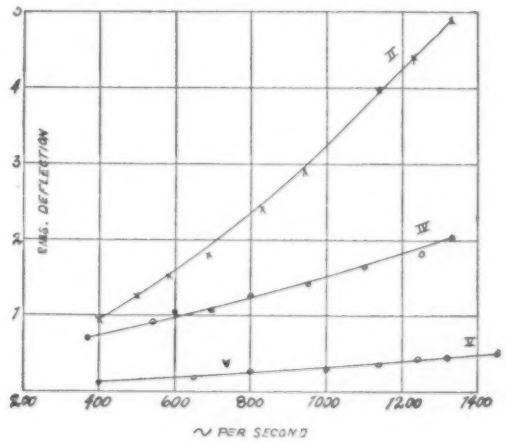

Fig. 3.

IV.		V.		IV.		V.	
F	*D*	*F*	*D*	*F*	*D*	*F*	*D*
374	0.71	408	0.10	952	1.41	1,143	0.33
540	0.92	645	0.18	1,143	1.65	1,210	0.39
597	1.01	727	0.20	1,250	1.82	1,240	0.42
656	1.06	800	0.24	1,333	2.08	1,324	0.45
798	1.28	1,000	0.29			1,450	0.50

A piece of music wire 0.027 cm. in diameter and about 0.8 cm. long gave the results for tables VI., VII. and VIII., all taken on different days and plotted, as Δ, x and o respectively in curve VI., Fig. 4. The current through the solenoid was 0.15 ampere.

Two nickel wires 0.014 cm. in diameter and 0.9 cm. long were twisted together and substituted for the iron and steel used in the preceding cases. The data for tables IX. and X. were taken using 0.15 ampere and 0.10 ampere respectively through the solenoid. Data were also obtained using 0.05 ampere but the deflections were quite small, though the variation with increase of frequency seemed

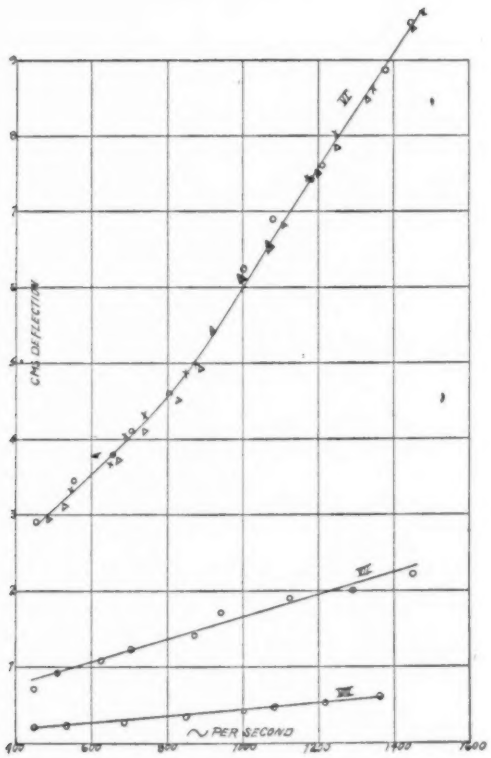

Fig. 4.

VI.		VII.		VIII.	
F	*D*	*F*	*D*	*F*	*D*
488	2.93	548	3.32	467	2.90
526	3.12	645	3.67	554	3.46
666	3.70	690	4.01	656	3.80
741	4.11	741	4.32	702	4.13
833	4.50	851	4.85	800	4.61
889	4.91	879	5.00	1,000	6.28
920	5.39	1,000	6.10	1,080	6.92
1,000	6.10	1,066	6.52	1,180	7.43
1,066	6.53	1,176	7.41	1,380	8.90
1,111	6.80	1,250	8.02	1,450	9.50
1,194	7.50	1,356	8.59		
1,250	7.85	1,481	9.60		
1,333	8.48				
1,450	9.48				

quite regular. Tables IX. and X. are plotted in curves VII. and VIII., Fig. 4.

The extremely high sensibility of this type of machine is readily seen from curves I., Fig. 2 and VI., Fig. 4. Taking curve I., for example, we have over 10 cm. deflection for 1,300 cycles per second. A decrease or increase in this frequency of .05 per cent. if it persists for a few seconds, is easily detectable, assuming the galvanometer at one meter distance. The number of revolutions of the inductor's shaft can of course be obtained with the same accuracy. By using a galvanometer of higher sensibility and more junctions of higher E.M.F. it seems quite probable that these sensibilities can be materially increased.

The flatness of curves III., IV. and V. indicate the desirability of keeping the magnetizing force as small as possible if a frequency meter having a uniform scale and a wide range is desired. It is quite possible that some magnetic alloy, or a combination of nickel and steel such as is used in Foley's[1] modification of the Marconi magnetic detector, can be found which will answer for even strong magnetizations. With the idea of low magnetization in mind an instrument has been constructed on the principle of the Boy's radio-micrometer. The bismuth and antimony strips at the lower end of the silver loop are soldered to a delicate vertical iron needle flush with their lower ends and extending upward about half their length. A rather long solenoid of a few turns of heavy silver wire extends for a few millimeters beyond either end of the bismuth and antimony strips. The small alternating current to magnetize the iron is sent through this solenoid. The instrument has proven sensitive to frequency, but no reliability can at present be attached to its indications. The chief troubles seem to be due to poorly-balanced junctions and to convection currents around the delicately suspended system.

The iron and steel curves for which the higher magnetizing forces are used indicate an increase in hysteretic loss per cycle as the frequency increases. Lloyd[2] in his discussion of the various methods used in the study of kinetic hysteresis shows the necessity of keep-

[1] Foley, PHYS. REV., 1904, Vol. XVIII., p. 349.

[2] Lloyd, Bulletin of the Bureau of Standards, Vol. V., No. 3, pp. 381–411.

ing the wave form constant. But even the gradual change in wave form which may have taken place in the current curve of the inductor used in this work will fail to account for the bending in curves obtained with the higher magnetizations. Furthermore, the maximum bending seems to occur always at frequencies between 600 and 1,000. It is also to be noted that the usual objection of insufficient regard for eddy currents can be urged less strongly in this case, inasmuch as both junctions are metallic and in the same alternating field, and while the induction in the iron is higher than in the other junction, yet for the high frequency alternations the impedance corresponding to a variation of flux with time would also be high for this same junction, thus giving a possible equality of eddy currents in the two opposed junctions for one frequency at least.

The superposition of rapidly alternating fields with oscillating fields of very high frequency has been tried with results so far in close agreement with those of Piola.[1] The writer expects to finish his work along this line, using widely different frequencies and magnetizations and report in the near future. Some typical results will now be submitted. A solenoid of 20 turns no. 18 B. & S. insulated copper wire 4.4 cm. long and 1.4 cm. in diameter was placed in the core of the fine wire solenoid previously described so that the two were concentric. The arrangement shown in Fig. 1 was disturbed only by the addition of this auxiliary solenoid and a source of oscillating current. With 0.28 ampere of oscillating current (frequency not definitely known, but very high) a deflection of 2.5 cm. was obtained from G_1. With 0.15 ampere alternating current, frequency 1,490, through the fine wire solenoid there was a deflection of 11.9 cm. With the two currents through their respective solenoids simultaneously and the frequencies unchanged, the deflection was 13.4 cm. For these results soft iron was used in the differential junction. With steel in the junction instead of iron, the sum of the deflections due to the two currents taken one at a time equalled the deflection when they were both acting as in the case above.

On account of the very satisfactory performance of the simple comparator shown in Fig. 1, the writer has devised a more elabo-

[1] Piola, Elettricista, Rome, 5, pp. 4-6, Jan. 1, 1906.

rate and convenient instrument, using the principle of differential thermo-junctions. Fig. 5 is a diagrammatic view of the connections. J_1 and J_2 are delicate, similar and opposed junctions of antimony and bismuth. H_1 and H_2 are similar heaters of a single short length of manganin wire of 0.9 ohm resistance. Slow motion screws permit delicate variations of distance between the junctions and their

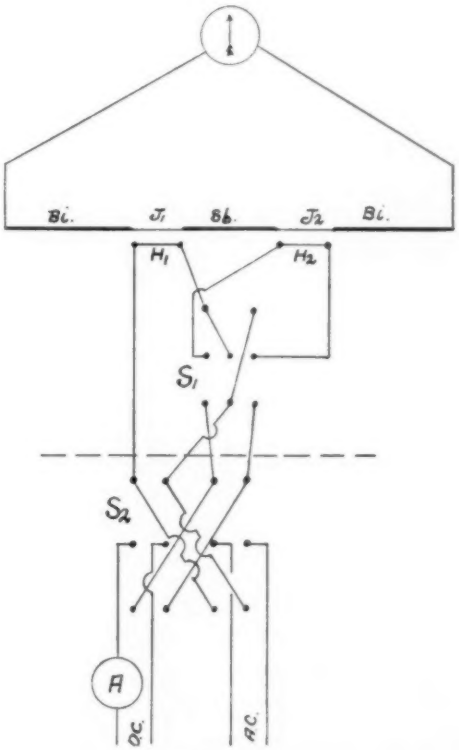

Fig. 5.

respective heaters. The double-throw switch S_1 has two poles above and three below, S_2 is four-pole and double-throw. The two heaters and their corresponding junctions have a heavy shield which is not shown in the diagram.

To compare an alternating current with a direct the very simple procedure is to put H_1 and H_2 in series by means of the switch S_1 thrown up. The current to be measured or a stronger one is then sent through the heaters by throwing S_2 down. The distance of H

and H_2 from the junctions are then adjusted until the galvanometer shows no deflection. Now since the same current through H_1 and H_2 causes no deflection of the galvanometer, equal currents through H_1 and H_2 will cause no deflection. Then by simply throwing S_1 down the alternating current is sent through H_1 and a direct current through H_2. When these two currents are equal the galvanometer shows no deflection. This equality is secured by adjusting the strength of the direct current. Then the current value is read from the standard ammeter A, or by means of a potentiometer and shunt if higher accuracy is desired.

The switch S_2 is shown only for the purpose of describing a reliability test. After obtaining a balance between a direct current of .5 ampere through H_2 and an alternating current through H_1 and noting that a variation of less than .1 per cent. in the measured current could be readily detected by the galvanometer, S_2 was thrown up in order to interchange the currents in the two heaters. Not the slightest motion of the galvanometer mirror could be observed. This test was for 60 cycles and gave a result which might have been predicted, for the capacities and self-inductance of the heaters are extremely small. In ordinary work not involving excessively high frequencies, S_2 may be dispensed with and, excepting the ammeter A, only the apparatus above the dotted line in Fig. 5 need be used.

The measurement of alternating voltages is quite as easy as that of currents when a non-inductive resistance is available. Two 150-volt Weston A.C.–D.C. voltmeters with a certified accuracy of .2 per cent. have been checked against each other and found to agree within .1 per cent. Currents as low as .5 ampere have been measured with an accuracy of .2 per cent., using the 0.9-ohm heaters.

This high sensibility combined with low resistance possesses obvious advantages over the relatively high resistance devices depending for sensibility upon the elongation of heated wires. From the published constants and sensibilities of our instrument makers it appears at once that to measure a given large alternating current with this new comparator the resistance of the manganin shunt can be decreased at least ten fold. Or, with a given amount of manganin with which to build a low resistance shunt, the current which can be measured has been increased by another order.

With the heater wires so short and so far apart, there is not the possibility of one circuit inductively influencing the other as in the case of instruments using a differential system of two long parallel wires stretched close together. Other serious objections which Thornton[1] has made to the use of parallel, stretched wire systems for comparators only need be mentioned as having no applicability to the differential thermo-junction method of A.C.–D.C. comparison.

It may finally be stated that the new instrument has the following useful features:

1. It is a zero instrument and completely under experimental control, involving as it does no dependence on calibration or instrumental constants.

2. It is extremely sensitive and as perfectly dead beat as the galvanometer with which it is used.

3. It has practically no self-inductance or capacity and is consequently independent of wave form and frequency, probably within very wide limits.

4. The two circuits are widely separated so that the inductive action of one upon the other is entirely negligible.

5. The resistance for high sensibility being quite low, it offers an accurate and inexpensive means for the measurement of very large currents for which the cost of low resistance shunts is an important consideration.

6. By the use of high resistance heaters its lower range can probably be extended to a milli-ampere at least.

7. It is quite simple and once assembled it is sturdy and easy to manipulate.

8. Its cost is trifling compared with that of the Kelvin balance or Siemens dynamometer for large currents.

The writer has under construction a differential thermo-galvanometer which this work has suggested, two opposed junctions in series being connected to the silver loop of the Boy's radio-micrometer. If necessary an insulating wall of mica can stand between the two junctions since the instrument is to be of the no deflection type.

I wish to thank Dr. A. W. Goodspeed for his continual interest

[1] Thornton, Electrician, 55, p. 796, Sept. 1, 1905.

and encouragement. The permission he has given me to draw freely upon the time of our mechanicians, Messrs. J. Sidle and F. Kalmbach, has removed many difficulties. I am also greatly indebted to Mr. J. Martin. The skill and appreciation with which he has attacked the mechanical details of the comparator helps to account for its addition to the instrumental equipment of the laboratory.

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CRYSTAL AND SOLID CONTACT RECTIFIERS.

By Alan E. Flowers.

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IN the Physical Review for March, 1909, there appeared an account of tests made by G. W. Pierce on a number of rectifiers, consisting of points resting on crystals. The crystals used were anastase, brookite and molybdenite. Of these the molybdenite was by far the most sensitive and possessed the largest current carrying capacity. In the accounts given of these experiments, and in the accounts published elsewhere, no definite and final conclusions were drawn as to the cause of the phenomenon, though various possible causes were investigated. Wishing to determine if possible the cause of this action a series of experiments was carried out and a description of these experiments and their results follows.

1. Search for and Selection of a Rectifier of Large Capacity.

It seemed that the first and most important thing to do was to find a rectifier of large current carrying capacity, as the rectifiers previously described were capable of delivering, at most, but a few milliamperes of rectified current.

Believing that high resistance was not necessary, the crystals having high conductivity were selected for study. The sulphides appear to be particularly high in conductivity and iron, zinc and

lead sulphides were tried for current carrying capacity and rectifying properties.

Of these, lead sulphide, galena, has by far the highest conductivity, and fortunately it showed in several samples not only large current carrying capacity, but very appreciable rectification. Single crystals showing sometimes 150 to 200 milliamperes of rectified direct current with a 3-volt (effective) alternating current supply. The crystals studied appeared to rectify about half the alternating current, though here it is well to emphasize the fact that different samples and different places on the same sample give widely varying results.

The samples of molybdenite examined required roughly a 21-volt effective alternating current supply to give 20 to 30 milliamperes of rectified direct current, and in such cases the alternating current ammeter indicated sometimes 500 or more milliamperes.

Believing that still larger capacities might be desired, two rectifiers were made by setting eight to ten samples of rectifying crystals in a little metal trough containing melted lead.[1] The lead on hardening made an absolutely certain contact with the body of the crystal and the metal trough constituted one electrode.

Pointed copper wires, in parallel, attached to a common return were set so that each touched a rectifying spot on one of the crys-

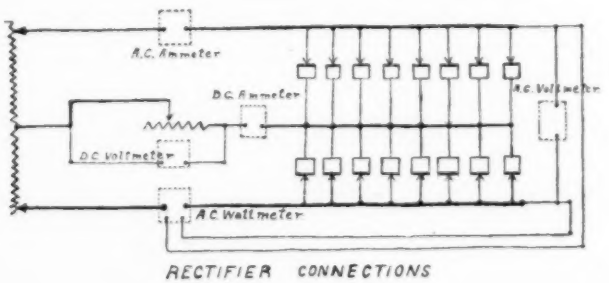


Fig. 1.

tals. (It is not essential to have but *one* point in contact with any one crystal, for if the rectifying surface be large enough, as many additional points may be used as there is room for on the rectifying surface.) See Fig. 1.

[1] Later I found that a similar method, using solder, had been used by L. W. Austin, Bulletin Bureau of Standards, Vol. 5, No. 1, p. 133, 1908.

These two rectifiers were connected in series-opposition across the secondary of a small transformer. One terminal of the direct current circuit was connected to the mid-point of a transformer and the other to the interconnection between the two rectifiers.

This rectifying set was left on the circuit all day without showing much change in its performance. A set of readings taken is shown in Table I.

From these results it was concluded that galena was the most promising crystal to study in order to find out the characteristics and cause of the rectification.

TABLE I.

Galena Crystal Rectifiers on Alternating Current. Frequency 60 Cycles. Observer A. E. F. Rectifiers A_{1-8} and B_{1-9}. Date of Test, April 10, 1909.

Time.	Alternating Current Input.			Direct Current Output.			
	Total Volts.	Total Amperes.	Total Watts.	Volts.	Amperes.	Watts.	Efficiency.
3:12 P. M.	3.75	.555	1.75	.56	.493	.276	15.7
10:12 P. M.	3.82	.560	1.75	.56	.483	.270	15.4

Note that B_1 was disconnected.
See Fig. 1 for diagram of connections.

2. CHARACTERISTICS OF THE GALENA RECTIFIER.

In order to analyze the action of the crystal it seemed best to try the effects on a direct E.M.F.

(*a*) *Volt Ampere Characteristics.*— A large number of observations were taken on different crystals. Of these the set of readings given in Table II. and plotted Fig. 2 are characteristic.

Discussion of Volt Ampere Characteristic.— The values given in Table II. and shown in Fig. 2 are fairly representative. A large number of other runs were made which showed similar results. In some cases 4.2 volts would give a current of .850 or more amperes from crystal to copper and only .025 amperes in the opposite direction.

The most important thing about these characteristics is that they show a point at which breakdown occurs. This point occurs at a fairly definite *current* for any *given set* of conditions of *setting*. In general, however, each crystal and each setting will have its own

breakdown point. This point may be 15 or more volts with 30 to 40 milliamperes flowing from point to crystal on fresh rectifying spots and may be as low as 5 volts and 90 milliamperes after several breakdowns. In general the higher breakdown voltages are reached when the current is small.

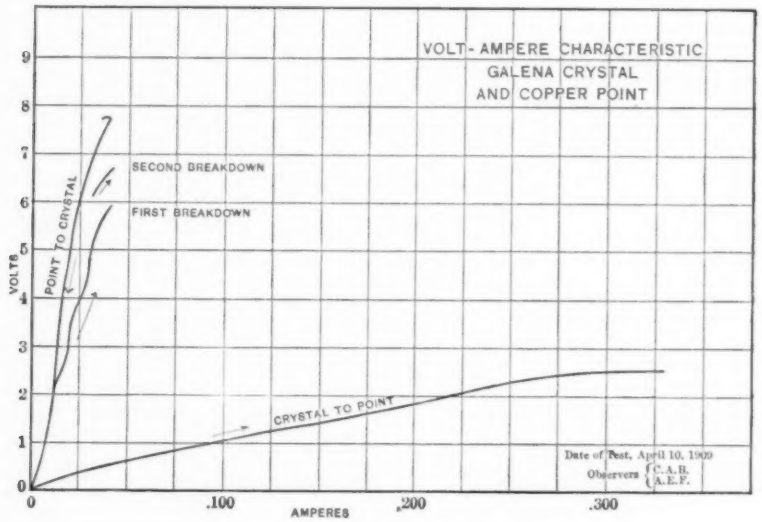


Fig. 2.

This breakdown point may occur at a higher voltage (and the same current) if the circuit be broken at the instant of partial failure and the crystal allowed to recover; while if the current is allowed to continue to flow after breakdown, partial or permanent destruction of the unidirectional conductivity or rectifying property will result.

The rectifying property is however not destroyed by the passage of considerable currents in the direction of crystal to point. In the case just cited 850 milliamperes did not destroy the rectifying property of the crystal.

Excessive current in *either* direction will break down and partially or permanently destroy the rectification, and breakdown occurs at a lower voltage when the current is reversed at each value of impressed voltage than it does when the voltage is simply increased in the direction of least current, *i. e.*, with the point as anode.

TABLE II.

Volt Ampere Characteristic of Galena Rectifier. Rectifier C, Copper Point Anode. Observers A. E. F. and C. A. B. Date of Test, April 10, 1909.

Time.	Volts.	Amperes.	Time.	Volts.	Amperes.
2:35 P. M.	.824	.005	Current was off for a short time while arranging for higher voltage.		
	1.492	.010			
	2.230	.013	2:35 P. M.	6.98	.026
	2.500	.016		7.58	.028
	2.704	.018		7.86	.030
	3.060	.019		8.36	.034
	3.288	.020		8.57	.039
	3.804	.023		8.96	.039 — Breakdown occurred at this point.
	3.900	.024			
	4.008	.025		6.64	.055
	4.292	.028		4.16	— ?
	4.472	.029	Breakdown occurred again. Resealing began at this voltage.		
	4.780	.030			
	4.960	.031		7.72	.035
	5.180	.032		7.76	.039
	5.308	.033		7·72	.039
	5.468	.035		7.38	.035
	5.940	.039		6.90	.030
At this point the crystal resistance seemed to break down, allowing several hundred milliamperes to pass. The voltage was reduced till resealing occurred and then raised.				5.69	.026
				5.39	.020
				3.36	.015
				1.83	.010
	6.104	.031		.72	.005
	6.36	.034	Copper Point Kathode.		
	6.44	.035			
	6.58	.037		.16	.009
	6.66	.039		.272	.018
	6.66	—		.32	.020
Breakdown occurred again and the voltage was reduced till resealing occurred and then raised again. The following readings were taken to prove that the passage of current in the previous runs had not spoiled the rectifying properties of the crystal.				.416	.030
				.60	.049
				.72	.052
				.944	.090
				1.12	.110
				1.28	.130
	1.92	.005		1.408	.143
	5.80	.020		1.68	.179
	6.36	.023		1.792	.196
	7.17	.028		1.872	.203
	7.48	.030		2.065	.229
	7.64	.030		2.25	.241
	7.78	.032		2.275	.257
	7.93	.033		2.41	.276
	7.98	.033		2.52	.310
	0	0		2.56	.329

Breakdown seems to be due to the heat produced. When viewed under a high-power microscope the point can be seen to sink into the body of the crystal, the surface bending and cracking like thin ice over mud, and little globules of molten material ooze through the crevices.

The point leaves a hole in the surface visible to the naked eye and breakdown carried to this extent is permanent.

Even before the final breakdown is reached one can perceive partial breakdown occurring and immediate resealing, resulting often in a decreased current. This is shown admirably in the varying curvature of the volt-ampere characteristic from point to crystal in Fig. 2.

The fact that the reactions are taking place at the surface or in a surface film is shown strikingly by measuring the potential difference between the copper point and the surface of the crystal near the point. It is found that practically the whole E.M.F. impressed on the rectifier is required for the potential difference between the copper point and the crystal. For example, a drop of 5.95 volts was measured between the copper point and the nearest spot on the crystal surface when the total voltage impressed on the rectifier was 5.98 volts.

(*b*) *Effect of the Material of the Point.* — Having found a crystal that was capable of rectifying large currents the next thing done was to try to determine whether the effects observed were in any way dependent on the material of the point used. Volt-ampere characteristics were taken with points of copper, steel, platinum, lead, zinc, aluminum, brass, solder and graphite.

The characteristics were similar to the ones illustrated in Fig. 2 and Table II., and apparently the material of the point made no difference in the rectification. Small differences, however, such for instance as those which might be caused by variation in heat conductivity of the points, might easily be masked by the considerable variations in the volt-ampere characteristics of different crystals.

(*c*) *Effect of the Shape of the Point and the Ratio of Contact Areas.* — The rectification did not seem to be affected in any way by change of shape of point. The steel needle, point end, seemed to show the same results as the eye end and sharp and blunt ends of copper

wire points seemed to give equally good results. Even the substitution of a globule of mercury about 3 mm. in diameter or of melted lead dropped on the crystal surface and allowed to harden in place so that it made intimate contact with a large area of the surface gave good rectification. It is unusual to find a large surface showing uniformly good rectification, and if any *part* of the globule touches a spot that does not rectify the effect of the rectifying surface is masked or lost.

For comparison a copper point was set on the rectifying surface near the globule of lead; the other contact to the crystal being made by a lead setting such as has already been described. The results of these tests are given in Table III.

TABLE III.

Comparison of Point Contact and Large Area of Contact on Rectification.
Observer A. E. F. Date of Test April 30, 1909.

Volts. Crystal to Point.	Amperes.	Resistance, Ohms.	Ratio of Resistance.	Remarks.
— 4.00	.003	1333		
+ 4.00	.190	21.05	.01577	Copper point under very light pressure.
— 4.00	.012	333		
+ 4.00	.192	20.8	.0625	
— 1.00	.023	43.5		Connection made to globule of lead.
+ 1.00	.186	5.38	.01236	
— 1.00	.049	20.2		Readings taken after several reversals.
+ 1.00	.216	4.63	.229	
— 2.00	.100	20		Current decreasing.
+ 2.00	.600	3.33	.1663	Current increasing.
— 3.00	.210	14.28		Current increasing.
+ 3.00	.980	3.06	.214	Current increasing. Breakdown finally.
— 1.00	.044	22.7		Contact made with crystal by a globule of mercury.
+ 1.00	.118	8.57	.373	
— 1.00	.039	25.6		Mercury globule contact on rectifying surface and copper wire point on non-rectifying crystal for the other connection.
+ 1.00	.088	11.36	.443	

The use of a large area of contact to the rectifying surface of the crystal does not destroy the rectification.

When the lead or mercury globule was used to make connection to the rectifying surface the substitution of a copper wire point in contact with the non-rectifying surface of the crystal instead of the usual lead setting had no very great effect on the rectification.

Finally a crystal was selected which showed good rectifying properties. This was mounted in a lead setting and the crystal cut down to a point. This pointed crystal was then set in contact with a large block of metal and also in contact with another crystal and it was found that rectification was produced as before.

The results of this test are given in Table IV.

TABLE IV.

Effect of Cutting the Rectifying Crystal Down to a Point. Contact Pressure 200 gms.
Observer A. E. F. Date of Test April 25, 1909.

P. D. Volts.	Current, Amperes.	Resistance Ohms. To Crystal.	Resistance Ohms. From Crystal.	Ratio of Resistances.	Remarks.
−4.00	.010	400			Crystal *E*, touching a steel
+4.00	.066		60.6	.151	hack saw blade.
−4.00	.010	400	100		Crystal *E*, touching a zinc
+4.00	.040			.25	plate.
−4.00	.020	200			Crystal *E*, touching lead
+4.00	.105		38.1	.191	plate.
−4.00	.011	363.5			Crystal *E*, touching brass
+4.00	.091		44.0	.1209	plate.
+4.00	.004	1,000			Crystal *E*, touching a recti-
−4.00	.004		1,000	1	fying galena surface.
−4.00	.012	333		.0333	Crystal *E*, touching a non-
+4.00	.360+		11.1		rectifying galena surface.
At this voltage the current continues to increase till breakdown occurs.					
+4.00	.074		54.1		Crystal *E*, touching alumi-
−4.00	.011	363.5		.1486	num plate.
+4.00	.070		57.2	.1715	Crystal *E*, touching brass
−4.00	.012	333			plate.
−6.35	.030	212		—	
−6.18	.074	83.5	—		
At this point breakdown is beginning.					
−5.3	.590				
Breakdown is complete.					

(*d*) *Effect of Mechanical Pressure and Mutilation of Surface.*— When a good rectifying surface has been found the copper point may be pressed against the rectifying surface with very considerable force without impairing the rectification and with a great increase in conductivity (in both directions). However, if the pressure is high and the voltage is carried to the breakdown point the breakdown of the spot touched is likely to be permanent. The point sinks into the crystal, or perhaps a hole is melted in the crystal and the point sinks into this hole.

It was found that a moderate pressure (about 200 grams) was sufficient to give steady contact and in most cases this was the pressure used.

A crystal having a rectifying surface was often found to have other rectifying surfaces underneath and parallel when the layers were split off, but scratching or scarring a rectifying surface usually spoiled more or less completely its rectifying properties.

(*e*) *Effects of Frequency.*— No attempt has as yet been made to determine a relation between per cent. rectification and frequency; however, the following significant tests bearing in a general quantitative way on this were carried out. A rectifier consisting of a copper point and galena crystal was connected in series with a fairly sensitive galvanometer and a telephone receiver. Speaking into the receiver caused large deflections of the galvanometer and loud continued vowel sounds would throw the galvanometer off scale. The sensitiveness of this galvanometer is such that $.0286 \times 10^{-6}$ ampere gives 1 mm. deflection, so that the rectified current must have exceeded 7 microamperes.

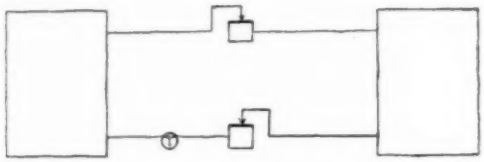
The frequency of telephone currents is of the order of 1,000 cycles per second.

Another arrangement of the rectifier was tried as a receiver for the electric waves produced by a static machine. The diagram of the connections is given in Fig. 3.

This arrangement gave galvanometer deflections of 3 to 4 cm. corresponding to a current of the order of a microampere with very high frequencies. Similar effects were obtained when using an induction coil as a source for the electric waves.

When using the galena crystal rectifier with a Tesla oscillator

large galvanometer deflections were obtained, but here a different type of rectifying effect also occurs between the point and any large conducting surface, such as the lead setting, a piece of brass, etc. That the type of rectification obtained with the Tesla oscillator is entirely different from the type of rectification here studied is made more clear by the results of the tests on the effect of electrical dis-



RECTIFIER AS WIRELESS RECEIVER

Fig. 3.

charges through gases. This type of rectification appears to be similar to the effects observed by

Tamm, Drude's Annalen, VI., p. 259, 1901,

Precht, Wiedemann's Annalen, XLIX., p. 150, 1903,

Himstedt, Wiedemann's Annalen, LXVIII., p. 294, 1899,

which show differences in the minimum voltage for discharge and in the conductivity of the gas between a point and a metal plate. The effects may be due to rectification at the point of connection to the oscillator where both brush discharge and sparking may be occurring and not at all at the point and crystal.

(*f*) *Effect of Electrical Discharges.* — A crystal, D_5, was mounted and connected to a storage battery with meters to measure the voltage impressed on the rectifier and the current flowing from the battery.

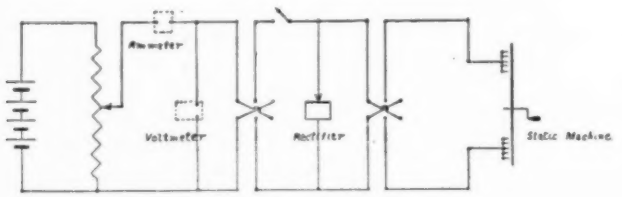


Fig. 4.

A static machine was then so connected that the static discharge could be sent in either direction through the rectifier as illustrated in Fig. 4.

It was found that the static discharge could be passed continuously in either direction through the rectifier, whether the rectifier were connected to or disconnected from the battery, without affecting the conductivity of the rectifier or its rectifying properties. However, it was found that if the static discharge produced a spark anywhere in its path the effect for any connection of battery or static discharge is to increase the conductivity in either direction and to spoil the rectifying property.

(*g*) *Effect of Heat.* — The determination of the effect of heat is complicated by the fact that the current that flows when the determination is being made may cause local heating and so intensify and mask the general effect. There may be also electro-chemical effects.

In general, however, breakdown occurs at a lower voltage with an increase of temperature and the conductivity rises with increase of temperature, but the increase is greater for the direction of least conductivity so that the per cent. of rectification decreases with increase of temperature.

The rectification disappears at a temperature of about 270° C., but is partially regained on cooling, though the parts of the crystal which carried no current are apparently not affected by heating to 270 or 280° C. under oil and then cooling. The effect of the passage of current through the rectifier while it was cooling is given in Table V.

TABLE V.

Effect of Continued Passage of Current on Rectifier while Cooling.

Volts.	Amperes.	Resistance.		Ratio of Resistances.
Cooling without current.				
— 1	.039	25.68		
+ 1	.074		13.52	.527
Cooling with current flowing from point to crystal.				
— 1	.014	71.5		
+ 1	.051		19.61	.2745
Cooling with current flowing from crystal to point.				
+ 1	.013		77	.539
— 1	.007	142.9		
+ 1	.020		50	
— 1	.008	125		.400

It is worth while noting also that even at 280° C. there is a tendency for the current, if from point to crystal to decrease, and if from crystal to point to increase, if the current is allowed to flow.

It is especially significant that the rectifying property is not entirely destroyed by the continued passage of current in either direction while cooling under oil from this temperature though the rectification is best when the current is allowed to flow from point to crystal.

Heating the rectifier in air in a closed vessel gave similar results up to 270° C., but at slightly higher temperature, 300–320, the difference in conductivity was regained. Here it often appeared that the difference in conductivity was not obtained till an appreciable time had elapsed. At 345° C. the difference in conductivity again decreased, and on cooling there was a continued decrease in conductivity in both directions till the current flowing at 1 volt was less than a milliampere. Parts of the crystal not carrying current were not affected.

After the passage of current in either direction the rectifier if disconnected quickly and connected to a galvanometer, showed a feeble E.M.F. always in the same direction and this direction was always opposite to that required for the difference in conductivity. This appeared to be due to a thermo-electric E.M.F. in the same direction as that which was produced when the *crystal* was gently heated by a flame.

3. Artificial Production of Solid Contact Rectifiers by Chemical and Electro-chemical Treatment.

An attempt was made to produce rectification in samples of galena that did not rectify naturally, by subjecting the crystal to the action of various chemicals. Water and the conducting solutions and acids tried (*i. e.*, nitric acid, alcohol, potassium bichromate), if put on the crystal while the crystal was carrying current, usually increased the conductivity, caused breakdown and spoiled rectification. Although such results did not occur when the chemical treatment was not coincident with passage of current. The non-conducting acids and solutions tried (*i. e.*, conc. H_2SO_4, CS_2 and oils) seemed to have little effect.

The copper point was then covered with burning sulphur, the flame blown out and the remaining sulphur cooled quickly so that it was left in the amorphous state. A point so treated when set on a galena crystal surface that previously showed no rectifying properties would at first show little conductivity, but the passage of current first in one direction and then in the other produced in a few minutes a very fair degree of rectification *but in the opposite direction to that found in native crystals.*

A set of values is given in Table VI.

TABLE VI.

Rectifier Produced Artificially by Electro-chemical Treatment of Copper Point with Amorphous Sulphur.

Potential Difference, Crystal to Point.	Current.	Resistance.		Ratio of Resistances.
		Point to Crystal.	Crystal to Point.	
−1.00	.172	5.88	—	.209
+1.00	.036	—	27.80	

The treatment of a crystal that previously possessed no rectifying properties with amorphous sulphur and then with current, produced rectifying properties similar in all respects to the effect obtained when the copper point was treated.

The effect of the electro-chemical treatment with amorphous sulphur of the point set on metallic lead or the treatment of the lead surface itself was much more marked. Table VII. gives the results obtained with this rectifier.

It is worthy of note that this contact rectifier showed itself far superior in rectifying power to any of the crystal rectifiers so far tried, its resistance in one direction being .3 to .4 per cent. of the resistance in the other direction. Its breakdown point was reasonably high and it showed at one point the partial breakdown and immediate resealing or self-recovery found with the crystal rectifier.

The copper point of a rectifier produced in this way (by electro-chemical sulphur treatment while the point was in contact with a piece of lead) could be removed and set on a block of brass or other metal and be made to exhibit rectifying properties, but the rectifying film had to be produced first in contact with lead.

TABLE VII.

Copper-point Lead-block Contact Rectifier Produced Artificially by Sulphur Treatment.

Potential Difference, Lead to Copper. Volts.	Current, Amperes.	Resistance.		Ratio of Resistances.
		Copper to Lead. Ohms.	Lead to Copper. Ohms.	
−1.00	.002	500		
+ .62	.312		1.986	.00397
−1.00	.002	500		
+ .60	.404		1.485	.00297
−2.00	.003			
−3.00	.004			
−4.00	.022			
−5.00	.004			
−6.00	.008			
−7.00	.009			
−8.00	.010			
Breakdown.				

Attempts to produce the rectifying film by electro-chemical treatment of a copper point with sulphur while in contact with brass, failed to produce definite results, though good results were obtained by setting the copper point, after being treated with sulphur, on a small block of graphite.

4. APPLICATIONS.

Rectifiers of this type, when perfected, might be used in measuring telephone currents, and sound intensities, in detecting feeble electric waves and in charging small storage batteries.

5. DISCUSSION OF THE RESULTS AND CONCLUSIONS.

It is worth while noting that other workers in this field have emphasized the thermo-electric potential difference of the two materials of the contact rectifier or have emphasized the difference in heat and electric conductivity of the two materials.

The rectification cannot be due to thermo-electric forces, since they have neither the magnitude nor the direction necessary. The rectification cannot be due to difference in conductivity of the two materials in contact, because graphite, which has a higher specific resistance than the lead sulphide, could be used for a contact making

point. There are only two possible explanations: One is that the film, when once formed, has unidirectional conductivity. This seems improbable in solids. The other explanation is that the film is produced in an infinitesimally short length of time by the passage of a minute quantity of electricity.

The tendency of the resistance to rise when current is flowing to the crystal, the direction of highest resistance, and to fall when the current is flowing from the crystal, seems to give color to this view. The sluggishness of change of resistance sometimes observed at high temperatures and with some crystals also tends to strengthen the probability that the rectifying film is re-formed electro-chemically with each reversal.

If the film is re-formed at each reversal, then there must be a definite quantity of electricity, however small, required for the production of the film, and one should find that the rectification is less perfect for very small currents or for very high frequencies. The facilities for determining this effect are not now at my disposal, but I trust to see this point settled in the near future.

The results of the tests described seem to justify the following conclusions:

1. The rectifying action is produced at the surface, or in a surface film, and is not due to the point employed.

2. The rectification is not an effect of the sharpness of the point or the ratio of contact areas.

3. The rectifying film will break down at a moderate voltage (6 to 15 volts) which is higher the smaller the current.

4. The rectifying film can be destroyed by the action of the electric current.

5. The feeble evanescent E.M.F. existing after the passage of current in either direction through the rectifier is insufficient to account for the rectification, and its direction is opposite to that which would be required for rectification.

6. There is apparently no difficulty in rectifying currents of very high frequency.

7. The rectification disappears at high temperature, *i. e.*, about 270° C., but is partially regained on cooling.

8. The rectifying film can be produced artificially by *electro-chemical* treatment with amorphous sulphur.

9. The rectification is probably due to the electro-chemical formation of a resisting film at each reversal.

It is a pleasure to acknowledge here the assistance rendered in various ways by a number of my colleagues.

UNIVERSITY OF MISSOURI,
June 22, 1909.

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ABSORPTION OF GASES BY THE ANODE IN A GLOW CURRENT.

By V. L. Chrisler.

THE first quantitative measurements on the absorption of gases under a glow current appear to have been made by Mey[1] who used a sodium-potassium alloy as cathode for purifying helium. Riecke[2] also passed a current for several hours through an atmosphere of nitrogen and measured the decrease in gas pressure. Neither of these observers found any electrolytic relation between the rate of absorption and the electric current. Skinner,[3] however, found later that most fresh metals when used as cathode with a glow current in helium or argon gave off hydrogen for a few minutes at the rate required by Faraday's law for electrolytes, while in hydrogen the pressure remained practically constant. He explained the absence of an increase in pressure in hydrogen by assuming that the cathode gave off gas at the same rate in hydrogen as in helium but that the anode absorbed it for a time at an equal rate. From experiments performed in a nitrogen atmosphere[4] he concluded that it also was absorbed by the anode for a time at the same rate as hydrogen.

The object of the present investigation was to obtain by a direct test the rate of absorption of these gases by the anode. The plan followed was to use a cathode which had previously been sufficiently freed of its content of gas so that it contributed nothing appreciable to the change in pressure. Then the effect of absorption at the anode could be directly obtained from the decrease in the pressure of the gas in the system.[5]

[1] K. Mey, Ann. d. Phys., Vol. 11, p. 127.

[2] E. Riecke, Ann. d. Phys., Vol. 15, p. 1003.

[3] C. A. Skinner, Phys. Rev., Vol. 21, p. 1; Phys. Zeitschr., 6, p. 610; Phil. Mag., Vol. 12, p. 481.

[4] C. A. Skinner, Phys. Rev., Vol. 21, p. 169.

[5] This plan had already been tried in this laboratory by Mr. Kiesselbach, who used several of the common metals with a limited degree of success. He found that they absorbed hydrogen from one to two minutes at the rate required by Faraday's law.

After first testing silver as anode, which was found to absorb hydrogen from two to four minutes at about the calculated rate, the alkali metals, sodium, potasssium, and a sodium-potassium alloy, were tried with marked success in a hydrogen and also in a nitrogen atmosphere. Finally mercury was tested, giving positive results in hydrogen but no absorption in nitrogen. In a few isolated cases out of a large number of tests helium was also absorbed by a sodium and by a mercury anode at double the rate (by volume) that hydrogen is under the same current. In most of the tests, however, practically no absorption of helium was obtained.

APPARATUS.

A battery of small storage cells served for producing the glow current, which was regulated by a cadmium iodide and amyl alcohol resistance, and measured by a Weston milliammeter. The gases used were hydrogen, nitrogen and helium. Hydrogen, obtained from aluminium in a solution of potassium hydrate, was, after careful drying by phosphorus pentoxide, stored in a flask which had stood evacuated for a time sufficient to free it of occluded gases. Nitrogen was obtained by passing air through phosphorus vapor, through a wash bottle containing a potassium hydrate solution, and finally through a tube containing phosphorus pentoxide. After drying, it was also stored in a flask freed of occluded gases. The helium used was obtained in sealed glass bulbs from Messrs. Thomas Tyrer & Co., London, from which it was transferred to another provided with a stop-cock, through which connection with the system was afforded.

In the vacuum system all ground joints and stop-cocks were lubricated with a mixture free of hydrocarbons. Phosphorus pentoxide was used as a drier. Evacuation was produced by a Barr and Stroud mercury pump. The pressure was measured by a McLeod gauge — one millimeter on the scale representing .01 mm. mercury pressure. The probable error in any measurement was not greater than .005 mm. although a maximum error of .01 mm. might be made.

EXPERIMENTAL RESULTS.

Silver as Anode. — The form of tube used for testing a silver anode in hydrogen is shown in Fig. 1. It was of glass provided with

the main electrodes A (the anode of silver) and C (the cathode of nickel), and an auxiliary electrode B of aluminium wire. The cathode was chosen of nickel as this metal was found to give off as cathode less gas than the others. Both electrodes A and C were made removable and were always carefully polished before mounting for a test. These preparations having been made the tube was evacuated and helium, to a pressure of about two millimeters of mercury, admitted. A very strong glow current was then passed simultaneously from A and C as cathodes to B as anode until the increase in gas pressure ceased. This cleaning current was made sufficiently intense to drive off the outer surface of the silver thus leaving a fresh surface which had been unexposed to other than the helium atmosphere. The success of all tests depended upon this treatment.

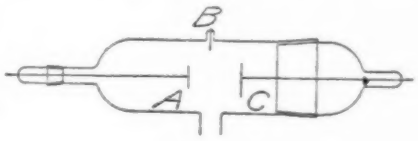

Fig. 1.

Having cleaned the electrodes in this manner the gas was quickly pumped out and hydrogen admitted. There was with all the metals tested a small gradual decrease in pressure of this gas for a time after admission arising from the absorption by the clean electrode and the metal deposited on the walls of the tube.[1] The nickel electrode, however, did not appear from the tests to absorb any of this gas. When the pressure became constant the electric current was started, with the nickel as cathode and the silver as anode. The change in gas pressure with time, under a current of about one milliampere, for three different tests is given in Table I. and plotted in Fig. 4. In the plot the straight line represents the decrement in gas pressure calculated from the electrochemical equivalent of hydrogen (104×10^{-7} gr. per coulomb), its density at room temperature and 76 cm. pressure (83×10^{-6}), and the volume of the tube (382 c.c.). It is to be noticed that for two minutes after the current started the decrease in pressure approximated closely the calculated value, from which it is to be concluded that with each atom of hydrogen absorbed is associated a negative charge of the same magnitude as that carried in solutions. When the surface of the metal

[1] See W. Heald, PHYS. REV., Vol. 24, p. 269.

becomes saturated with the gas it is to be expected that the absorption will stop, or, as suggested by Skinner (*l. c.*), that uncharged gas will leave the metal as fast as the charged gas enters it. In the case of silver this condition was reached in a short time and absorption ceased abruptly.

Soda Glass as Anode. — The investigation was next directed to sodium with the hope of obtaining absorption of longer duration. It was introduced into the tube first by electrolysis of the glass according to Warburg's[1] method. The tube used (shown in Fig. 2) was made of soda glass and furnished with a nickel electrode *C* which in the tests served as cathode, and an auxiliary electrode *B*. The end of the tube was immersed in a cup containing sodium amalgam through which the tube was heated by a Bunsen flame to a temperature such that the current could be passed through the glass and from it as anode through the gas to the cathode. Before the test the cathode was freed of gas by passing a current from *B* as anode. To obtain first a deposit of sodium inside the tube a glow current was passed through the glass and a helium atmosphere for a time before testing for the absorption of hydrogen. Having obtained the desired deposit the helium was removed and hydrogen admitted. The results from the different tests were very inconsistent, and, since the temperature was sufficient to deposit sodium (by vaporization) over the inner walls of the tube and on the other electrodes, the suggestion was that the natural absorption (or evolution of gas) by this deposit masked the effect of absorption by the anode.

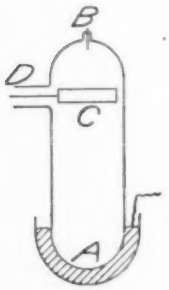

Fig. 2.

Soda Glass as Cathode. — An interesting thing was noticed in connection with the tests of the conducting glass. When it was used as cathode in helium the gas pressure increased at a rate about equal to that calculated on the assumption that hydrogen was the carrier of the current from the cathode. The spectrum, however, indicated that the gas freed from the glass was nitrogen. Representative observations of the increase in gas pressure with time are

[1] E. Warburg, Ann. d. Phys., Vol. 40, p. 1.

given in Table II. Like those obtained by Skinner[1] with a carbon electrode these results indicate that a negative electron is associated with each atom of nitrogen leaving the glass cathode. This is consistent with the fact that the chemical valence of nitrogen is in some cases unity. On reversing the current the gas which had been freed was reabsorbed (presumably by the glass), and on two occasions practically all of the helium atmosphere was absorbed with it.

Sodium Anode in Hydrogen. — To test the absorption of sodium at lower temperatures than the electrolytic method of introduction

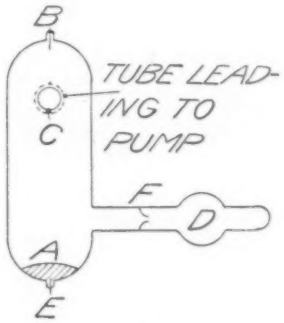

Fig. 3.

allowed, metallic sodium was introduced directly into the tube, the form of which is shown in Fig. 3. The sodium was first placed in the bulb *D* and the end of this side tube sealed. During continuous operation of the evacuating pump the sodium was then

TABLE I.

Silver Anode in Hydrogen. Volume 382 *c.c.*

Duration Current (Min.).	Current 1 ma. Initial Pressure 2.04 mm.		Current 1 ma. Initial Pressure 1.7 mm.		Current .9 ma. Initial Pressure .94 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.02	.015	.015	.015	.01	.0135
2	.035	.03	.025	.03	.02	.026
3	.035	.045	.025	.045	.03	.040
4	.04	.06	.02	.06	.04	.054
5	.045	.075			.03	.067
6	.05	.09				

[1] C. A. Skinner, PHYS. REV., Vol. 21, p. 168.

TABLE II.

Glass Cathode in Helium.

Duration Current (Min.).	Current 1 ma. Volume 277 c.c. Initial Pressure 2.21 mm.		Current 1 ma. Volume 304 c.c. Initial Pressure 2.245 mm.	
	Pressure Increment.	Calculated Increment.	Pressure Increment.	Calculated Increment.
1	.02	.0207	.025	.019
2	.04	.041	.045	.038
3	.06	.062	.065	.057
4	.07	.083	.075	.076
5	.08	.104	.085	.096
6			.09	.113
7			.095	.132

TABLE III.

Sodium Anode in Hydrogen.

Duration Current (Min.).	Current 1 ma. Volume 263 c.c. Initial Pressure 1.84 mm.		Current .5 ma. Volume 263 c.c. Initial Pressure 2.165 mm.		Current 1.5 ma. Volume 268 c.c. Initial Pressure 1.6 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.025	.022	.02	.011	.03	.032
2	.05	.043	.03	.021	.055	.064
3	.07	.065	.04	.032	.08	.096
4	.095	.085	.045	.043	.11	.128
5	.115	.109	.05	.053	.13	.160
6	.14	.131	.06	.065	.16	.192
7	.16	.151	.07	.075	.18	.224
8	.18	.174	.08	.086	.21	.256
9	.205	.196	.085	.096	.23	.288
10	.225	.218	.095	.107	.25	.320
11	.25	.240	.01	.118	.27	.352
12	.27	.262	.11	.128	.29	.384
13	.29	.283	.11	.139	.31	.416
14	.31	.305			.33	.448
15	.33	.327			.34	.480
16	.345	.349				
17	.36	.371				
18	.38	.392				
19	.39	.414				
20	.41	.436				
21	.43	.458				
22	.45	.479				
23	.46	.501				
24	.48	.523				
25	.49	.545				
26	.50	.567				

melted and allowed to flow through the trap F into the bottom of the tube at A where electric connection with it was made by a platinum wire sealed in at E. Preliminary to testing for the ab-

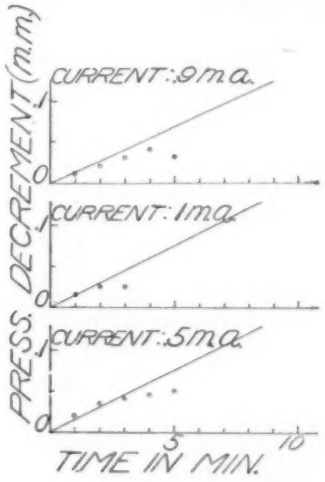

Fig. 4.

sorption of hydrogen the sodium and nickel C were freed of gas as before by using them as cathodes with a very strong glow current in helium.

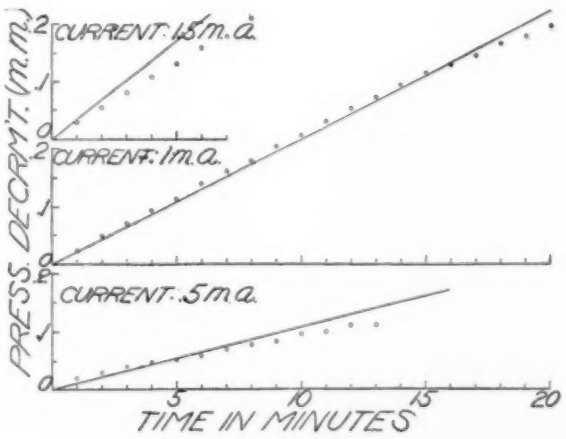

Fig. 5.

In Table III. and Fig. 5 is given, for the sodium as anode and nickel as cathode, the decrease in pressure of a hydrogen atmos-

phere with duration of current. Since absorption continued much longer than with a silver anode the initial pressure of the gas was taken at the end of the first minute to eliminate the effect of a change in temperature and the excess of initial absorption caused by starting the current. With .5 milliampere the absorption agreed with the calculated value for about ten minutes but stopped completely at the end of twelve minutes. With 1 milliampere it followed the calculated value for 15 minutes and continued slowly even after twenty-six minutes. With 1.5 milliamperes absorption according to the calculated rate lasted for only two minutes although it continued to absorb slowly for fifteen. The short duration in the last case is ascribed to a less favorable condition of the metal surface. It was noticed that during the absorption the surface of

TABLE IV.

Potassium Anode in Hydrogen. Volume 276 *c.c.*

Duration Current (Min.).	Current 1 ma. Initial Pressure 1.67 mm.		Current .5 ma. Initial Pressure 1.985 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.02	.021	.01	.0104
2	.04	.042	.02	.021
3	.06	.062	.035	.031
4	.075	.083	.045	.042
5	.095	.104	.055	.052
6	.115	.125	.065	.062
7	.135	.146	.08	.073
8	.155	.166	.09	.083
9	.170	.187	.10	.094
10	.19	.208	.11	.104
11	.21	.229	.125	.114
12	.22	.250	.135	.125
13			.145	.135
14			.155	.146
15			.165	.156
16			.175	.166
17			.185	.177
18			.200	.187
19			.21	.198
20			.22	.208
21			.225	.218
22			.235	.229
23			.235	.248

the sodium was blackened, indicating that sodium hydride was being formed. By using the sodium as cathode in a helium atmosphere it was found that this surface was almost entirely cleaned off under a strong current, and the disappearance accompanied by an increase in gas pressure slightly greater than the amount of hydrogen previously absorbed.

Potassium Anode in Hydrogen. — The form of tube used with the potassium, and the treatment of the metal, were the same as with sodium. The results of tests made under similar conditions are given in Table IV. and plotted in Fig. 6. Of these the .5 milli-

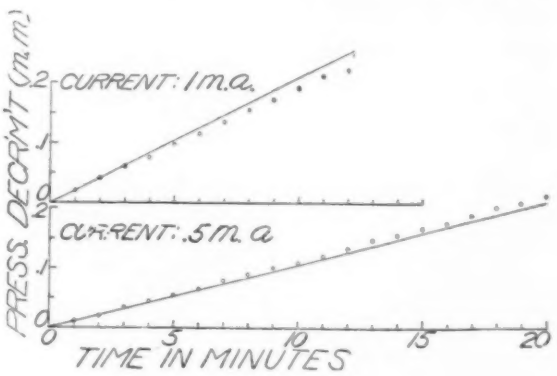

Fig. 6.

ampere gave the best results. The absorption followed the calculated value for twenty-two minutes and then suddenly stopped. With 1 milliampere absorption lasted for twelve minutes slightly below the calculated value. The test with 1.5 milliamperes gave no absorption, arising undoubtedly from an unfavorable condition of the surface of the metal.

Sodium-Potassium Anode in Hydrogen. — Sodium and potassium alloyed in atomic ratio was tested in the same form of tube and treated in the same manner as the two preceding metals. The results are given in Table V. and plotted in Fig. 7. With .5 milliampere the absorption followed closely the calcutated value for eight minutes, then suddenly stopped. With 1 milliampere it lasted about eleven minutes at a rate slightly below that calculated, while with 1.5 milliamperes agreement with the calculated value only lasted for about three minutes.

TABLE V.

Sodium-Potassium Anode in Hydrogen. Volume 273 c.c.

Duration Current (Min.).	Current 1 ma. Initial Pressure 2.375 mm.		Current .5 ma. Initial Pressure 2.24 mm.		Current 1.5 ma. Initial Pressure 1.455 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.02	.021	.015	.0105	.03	.0315
2	.045	.042	.025	.021	.06	.063
3	.06	.063	.035	.032	.09	.094
4	.08	.084	.04	.042	.11	.126
5	.10	.105	.05	.053	.13	.158
6	.12	.126	.06	.063	.145	.221
7	.14	.147	.07	.074		
8	.16	.168	.08	.084		
9	.18	.189	.08	.095		
10	.20	.21				
11	.22	.231				
12	.23	.252				
13	.24	.273				
14	.26	.294				
15	.275	.315				

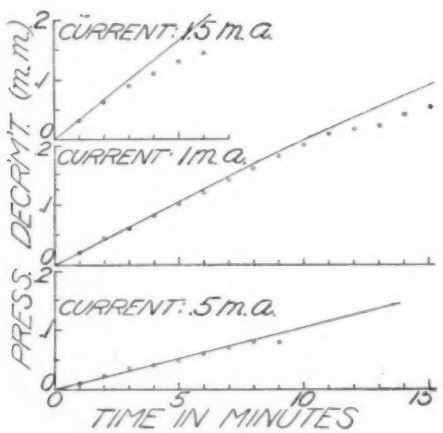

Fig. 7.

Mercury Anode in Hydrogen. — The form of tube used and the treatment of this metal was the same as with the alkali metals. The amount of hydrogen absorbed by the mercury before the current started was found to be much greater than in any other cases. In Table VI. is given a series of observations showing the gradual

TABLE VI.

The Absorption of Hydrogen by Mercury without Current. Volume 290 *c.c.*

Time (Min.).	Gas Pressure (mm.).	Pressure Decrement.
0	2.41	.0
1	2.4	.01
2	2.39	.02
3	2.38	.03
4	2.38	.03
5	2.365	.045
6	2.37	.04
7	2.365	.045

change in pressure of hydrogen after simply being admitted into the discharge tube. The results of its absorption by mercury as anode are given in Table VII. and Fig. 8. All tests show first an exces-

TABLE VII.

Mercury Anode in Hydrogen.

Duration Current (Min.).	Current 1 ma. Volume 295 c.c. Initial Pressure 2.03 mm.		Current 1.5 ma. Volume 290 c.c. Initial Pressure 2 83 mm.		Current 2 ma. Volume 290 c.c. Initial Pressure 3.18 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.03	.0194	.08	.0297	.11	.0396
2	.055	.039	.15	.059	.18	.079
3	.085	.058	.18	.089	.22	.119
4	.11	.078	.22	.119	.27	.158
5	.135	.097	.25	.149	.31	.198
6	.155	.116	.28	.178	.35	.237
7	.175	.136	.32	.208	.39	.277
8	.195	.155	.34	.238	.42	.317
9	.215	.175	.37	.267	.44	.356
10	.235	.194	.395	.297	.455	.396
11	.25	.214				
12	.27	.233				
13	.285	.252				
14	.30	.272				
15	.315	.291				
16	.33	.311				

sive initial absorption but later agree with the calculated rate. With 1.5 and 2 milliamperes agreement is found between the two minute and the eight and ten minute points, while with 1 milliampere it is between the three and fifteen minute points. The excessive initial

absorption which was so marked with mercury took place to a slight degree also with the alkali metals. With them, however, it occurred within the first minute after the current started so that the effect was eliminated by beginning observations after this time had elapsed. This excessive absorption during the first few minutes of current

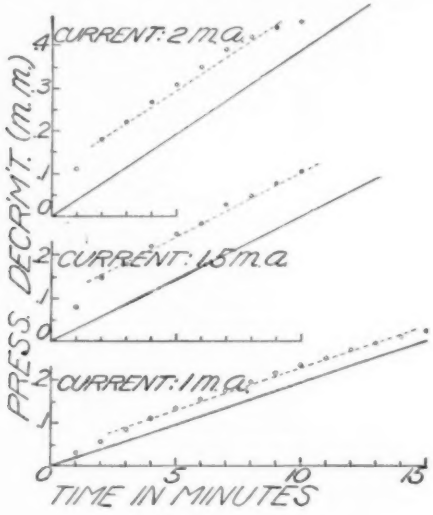

Fig. 8.

indicates that the metal will absorb more uncharged gas in a conducting state than when non-conducting. It is probably related to the fact noted by the writer[1] in an earlier paper that the photoelectric activity of a metal when placed in an atmosphere of hydrogen increased far more rapidly when the gas was conductive than when not.

Sodium Anode in Nitrogen. — The tests in hydrogen were followed by similar ones in nitrogen — the form of tube and treatment of the metal being the same. The results of the absorption of nitrogen by a sodium anode are given in Table VIII. and plotted in Fig. 9. The calculated value assumes that each atom absorbed carries the same charge as hydrogen. Absorption was not of as long duration as in the case of hydrogen. With .5 milliampere it agreed with that calculated for the first five minutes, then practically stopped; with 1 milliampere it agreed for the first four minutes and

[1] V. L. Chrisler, PHYS. REV., Vol. 27, p. 267.

TABLE VIII.

Sodium Anode in Nitrogen. Volume 295 c.c.

Duration Current (Min.).	Current 1 ma. Initial Pressure 3.65 mm.		Current .5 ma. Initial Pressure 2.41 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.02	.0194	.01	.010
2	.04	.039	.025	.019
3	.06	.058	.03	.029
4	.08	.078	.04	.039
5	.09	.097	.05	.049
6	.105	.116	.05	.059
7	.12	.136	.055	.068
8	.13	.146		

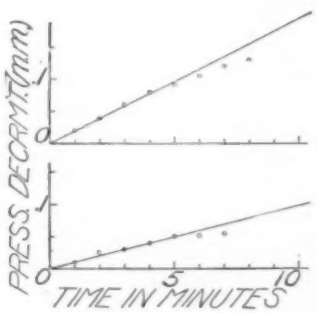

Fig. 9.

finally stopped at the end of eight minutes. A test with 1.5 milliamperes gave no absorption, probably owing to unfavorable surface conditions or possibly because the current density was too great.

Potassium Anode in Nitrogen. — The results showing the absorption of nitrogen by a potassium anode are given in Table IX. and Fig. 10. The duration of absorption was about the same as with sodium. The .5 milliampere gave the best results, the absorption agreeing closely with that calculated for five minutes and stopping entirely at the end of six minutes. With 1 milliampere the absorption followed the calculated value for four minutes, while 1.5 milliamperes agreed with that calculated for only two minutes.

Sodium-Potassium Anode in Nitrogen. — Results of absorption of nitrogen by this alloy are found in Table X. and Fig. 11. It lasted

TABLE IX.

Potassium Anode in Nitrogen. Volume 295 c.c.

Duration Current (Min.).	Current 1 ma. Initial Pressure .53 mm.		Current .5 ma. Initial Pressure 2.83 mm.		Current 1. ma. Initial Pressure 2.595 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.02	.0194	.01	.0097	.03	.029
2	.04	.039	.02	.019	.06	.058
3	.06	.058	.03	.029	.075	.087
4	.08	.078	.04	.039	.09	.116
5	.09	.097	.05	.049	.10	.146
6	.095	.116	.055	.058		
7			.055	.068		

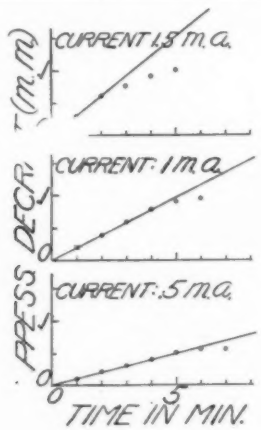

Fig. 10.

longer than with either sodium or potassium. With 1 milliampere the absorption was, for the first two minutes, greater than calculated, probably for the same reason that mercury absorbed an excessive amount of hydrogen for the first two or three minutes. Between the two and 15 minute points the rate agreed quite closely with that calculated. A current of .5 milliampere gave absorption which agreed fairly well with the calculated value for ten minutes while with 1.5 milliamperes it agreed for three minutes.

Glass, Sodium and Mercury Anodes in Helium. — It was noticed when depositing sodium by electrolysis of glass, using an atmosphere of helium, that the helium was sometimes absorbed. In sev-

TABLE X.

Sodium-Potassium Anode in Nitrogen. Volume 295 c.c.

Duration Current (Min.).	Current 1 ma. Initial Pressure 2.73 mm.		Current .5 ma. Initial Pressure 2.48 mm.		Current 1 5 ma. Initial Pressure 2.02 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.03	.0194	.01	.0097	.03	.0291
2	.07	.039	.02	.019	.06	.058
3	.09	.058	.04	.029	.085	.087
4	.11	.078	.05	.039	.11	.116
5	.13	.097	.06	.049	.13	.146
6	.15	.116	.07	.058	.14	.175
7	.17	.136	.08	.068		
8	.185	.155	.09	.078		
9	.205	.175	.10	.087		
10	.225	.194	.11	.097		
11	.24	.213	.115	.107		
12	.26	.233	.115	.116		
13	.275	.252				
14	.29	.272				
15	.315	.291				
16	.325	.311				
17	.345	.330				
18	.355	.349				

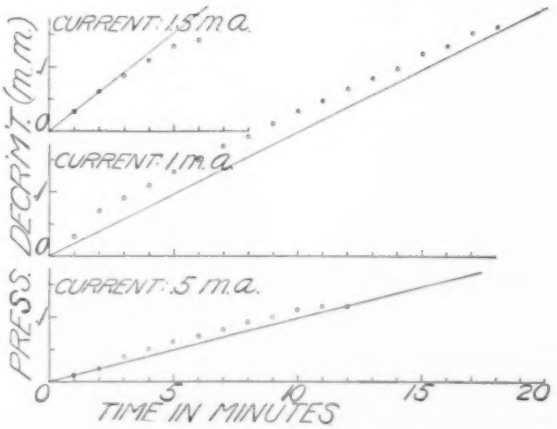

Fig. 11.

eral cases it was in only relatively small quantities, but in two others practically all of it was absorbed. The maximum rate of absorption in these two cases, which was attained a few minutes after the cur-

rent started, lasted for several minutes in approximate agreement with that obtained later with sodium and mercury anodes. Results obtained from these two metals are given in Table XI. and Fig. 12.

TABLE XI.

Duration Current (Min.).	Mercury Anode in Helium. Current 1 ma. Volume 295 c.c. Initial Pressure 2.845 mm.		Sodium Anode in Helium. Current 1 ma. Volume 297 c.c. Initial Pressure 1.05 mm.	
	Pressure Decrement.	Calculated Decrement.	Pressure Decrement.	Calculated Decrement.
1	.035	.0388	.04	.0386
2	.075	.078	.08	.077
3	.105	.116	.11	.116
4	.125	.155	.14	.154
5	.155	.194	.16	.193
6	.165	.233	.17	.232
7	.175	.273	.18	.270
8	.195	.310	.20	.308
9	.205	.349	.21	.347

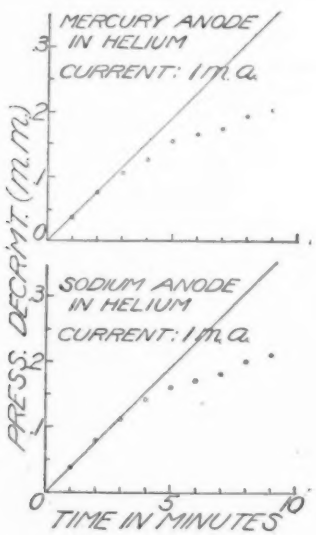

Fig. 12.

In the case of the sodium the helium used had been stored for several months in the bulb and hence might have been impure. That used with mercury, however, was fresh, having been transferred from the sealed glass bulb, in which it was obtained only a few

days before the test. In both cases the results agree with each other very closely, giving a decrement in pressure just twice that which would have occurred had the same current passed through an atmosphere of hydrogen instead of helium. The calculated value was obtained by assuming that helium is monatomic and that with each atom absorbed there was associated the same charge as with hydrogen and nitrogen. Close agreement is found for the first three minutes after the current was started. Several attempts were made later to reproduce these results, but without success, probably owing to the fact that proper surface conditions were not obtained. The rate of absorption in the successful experiments as well as the failures to obtain any absorption indicate that the effects did not arise from the presence of the common gases as an impurity in the helium, because with these absorption always took place and at a distinctly different rate.

To sum up, it has been shown by the foregoing tests that hydrogen is absorbed by sodium, potassium, a sodium-potassium alloy and mercury when used as anode in a glow current, at such a rate as to indicate that with each atom absorbed is associated a negative charge of the same magnitude as the positive carried by it in solutions. The same has also been shown with nitrogen, using sodium, potassium and a sodium-potassium alloy as anode; and in two cases, with helium, using sodium and mercury as anodes. The fact that nitrogen carries the same charge as hydrogen is consistent with the chemical valence of nitrogen, being in some cases unity. Rutherford[1] has concluded from his experiments that the α particle from radium is a helium atom carrying a positive charge of *twice* the magnitude of that carried by a hydrogen atom in solution, while the results given here indicate on the other hand that with the helium atom is associated a negative charge of the *same* magnitude as that carried by hydrogen.

I wish here to express my appreciation and thanks to Dr. C. A. Skinner, of the University of Nebraska, for his help and inspiration to me in this work.

THE BRACE LABORATORY OF PHYSICS,
UNIVERSITY OF NEBRASKA, LINCOLN,
June, 1909.

[1] E. Rutherford, Phil. Mag., p. 281, February, 1909.

CRYSTAL RECTIFIERS FOR ELECTRIC CURRENTS AND ELECTRIC OSCILLATIONS.

PART III. IRON PYRITES.

BY GEORGE W. PIERCE.

Reference to the Previous Work. — In the previous installments[1] of this paper the writer has given an account of some experiments on the rectification of small alternating currents and oscillations by certain conducting crystal substances (carborundum, anatase, brookite and molybdenite) in contact with solid metallic electrodes. During the progress of these experiments L. W. Austin[2] has independently made similar experiments on rectifiers consisting of silicon, carbon and tellurium in contact with steel or aluminum.

These experiments are of considerable theoretical and practical importance. The practical importance arises in the fact that the rectifiers have met with highly successful application as detectors for the electric waves of wireless telegraphy.

The present note describes an experiment on the rectification of alternating currents by iron pyrites in contact with copper. Iron pyrites is one of the substances originally investigated for unilateral conductivity by Ferdinand Braun[3] in 1874. In an account of his experiments Braun describes the phenomenon as follows: "With a series of natural metallic sulphides — copper pyrites, iron pyrites, galena, and copper antimony sulphide — I obtained in general the phenomenon that the current strength was different for different directions of the current; that this difference increased with increase

[1] G. W. Pierce, "Crystal Rectifiers for Electric Currents," Part I., PHYS. REVIEW, Vol. 25, pp. 31–60, 1907; Part II., ibid., Vol. 28, pp. 153–187, 1909, and Proc. Am. Acad., Vol. 44, pp. 317–349, 1909. See also Pierce, "A Simple Method of Measuring the Intensity of Sound," Proc. Am. Acad., Vol. 43, pp. 377–395, 1908.

[2] L. W. Austin, "Some Contact Rectifiers of Electric Currents," Bull. of the Bureau of Standards, Vol. 5, pp. 133–147, 1908.

[3] Ferdinand Braun, Pogg. Ann., Vol. 153, p. 556, 1874.

of the current, and that, on keeping the circuit closed, the current for that direction in which the resistance was smaller increased, while for the opposite direction it decreased." [1] The use of iron pyrites in Braun's original experiments was mentioned in the historical note in Part I. of the writer's account.

While in Braun's experiments, so far as can be judged from his data, the current in one direction was at most twice as much as the current in the opposite direction under the same voltage, the phenomenon described by the writer in the present series of papers is of entirely a different order of magnitude from that investigated by Braun. In the present experiments, with various crystal rectifiers, the current in one direction is often several hundred times as great (in one case 4,000 times as great) as that in the opposite direction under the same applied voltage.

The Specimens of Iron Pyrites. — Recently, Mr. Melville Eastham, of the firm of Clapp and Eastham of Boston, has had the kindness to present the writer with some specimens of iron pyrites that rectify much more strikingly than the specimens studied by Braun, and compare favorably with the results obtained by the writer with carborundum, anatase, brookite and molybdenite. One of Mr. Eastham's selected specimens was used in the present experiment.

The fragment of iron pyrites was held in a clamp holder (see Figs. 2 and 3 of Part II.), and a copper rod was brought into adjustable contact with an exposed portion of the pyrites. With a careful adjustment of the contact at particularly favorable spots of the pyrite the device is a very sensitive detector for electric waves.

Current-voltage Curves for Iron Pyrites. — The rectifying action of the pyrites is shown by the current-voltage curves of Fig. 1, in which the current through the rectifier is plotted against applied steady voltage. The upper curve gives the current when the voltage is applied in one direction; the lower the corresponding current when the voltage is reversed. As was the case with all the solid rectifiers thus far investigated, the writer finds that for some adjustments of the contact the greater current is from the copper

[1] Translated from Braun, l. c., p. 558. The change of intensity of the current with its duration, as described in the paragraph from Braun's account, does not appear in the present writer's experiments, except in cases in which the current is large enough to hea the contact.

to the crystal and for other adjustments (*e. g.*, with a different pressure or at a different point of the crystal) the greater current is from the crystal to the copper. With a fixed adjustment, however, and at a constant temperature the current under a given applied voltage is quite constant.

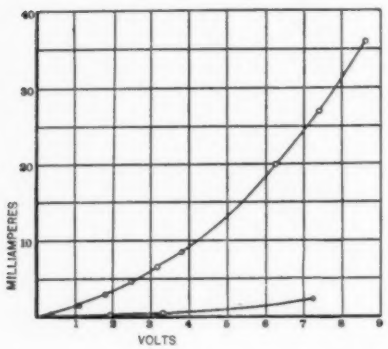

Fig. 1.

The Oscillogram for Iron Pyrites and the Oscillographic Apparatus. — Immediately after the current-voltage measurements plotted in Fig. 1 were made and with the adjustment of the contact of the rectifier undisturbed an oscillogram was taken of the current through the rectifier with a 60-cycle applied alternating voltage. This oscillogram is shown in Fig. 2. The oscillographic apparatus, which is described in detail in Part II. where several other similar oscillograms are reproduced, makes use of a Braun's cathode tube. Some small changes were made in this apparatus since the experiments of Part II. The most significant of these changes was a diminu-

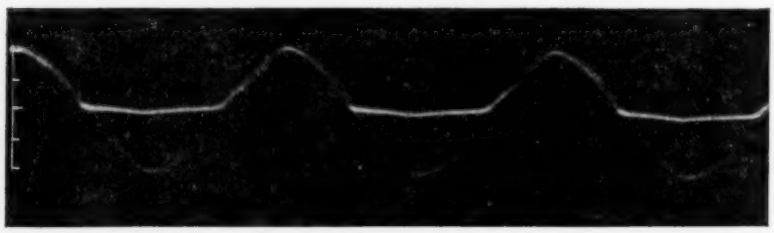

Fig. 2.

tion of the resistance of the coils used for deflecting the cathode beam. In the present experiment, these coils had a resistance of 43.2 ohms, while in the previous experiments the resistance of these coils was 436 ohms. The inductance of the coils in the present experiments was also less than that of the coils previously employed. The value of this inductance is determined below. The diminution of the inductance and resistance of the deflecting

coils, although resulting in a decreased sensitiveness of the oscillographic apparatus, carries with it a double advantage. The first advantage is that it improves the rectification by a given rectifier; for with a large series impedance in the rectifier circuit and with a given alternating voltage applied to the circuit the drop in voltage about the rectifier is small in the direction in which the current is large, on account of the large drop in the deflecting coils, while in the direction in which the current is small the drop in the rectifier is large because of the small drop in the deflecting coils. This has the effect of reducing the ratio that the large current bears to the small current. A second advantage of the diminution of the resistance and inductance of the deflecting coils of the oscillographic apparatus is that it reduces the masking effect that large series inductance and resistance have on any slight change of phase relations that might accompany the use of the rectifier.

Description of the Three Exposures. — In taking the present oscillogram, as in taking the oscillograms of Part II., three exposures were made on the film: First, an exposure of 20 seconds was made with the circuit open. This gave the axis of no current, as a light horizontal straight line through the picture. Second, with an applied alternating voltage of 5.78 R.M.S. volts and with the rectifier in the circuit an exposure of two minutes was made. This exposure gave the rectified cycle, consisting of an upper loop for a half period and a nearly straight line (with a slight downward depression) for the next half period. Third, the rectifier was replaced by an ohmic resistance (the "equivalent resistance") selected to make the maximum point of the excursion of the spot, as seen on the luminescent screen of the cathode tube, agree with the maximum point of the excursion obtained with the rectifier in circuit. This exposure (of one minute) gave the approximately sine-curve, which in Part II. was called the "voltage-phase cycle."

Data of the Oscillographic Apparatus. — The data of the oscillographic circuit in the present experiment were:

Maximum rectified current = 21.7 milliamperes,
Applied alternating voltage = 5.78 R.M.S. volts,
Equivalent resistance = 320 ohms,
Resistance of deflecting coils = 43.2 ohms.

The deflections of the luminescent spot were proportional to the current, and the iron of the deflecting coils was so feebly magnetized that it showed no hysteresis. As in Part II., the inductance of the coils was calculated from the amplitude of the voltage-phase cycle as follows: The maximum applied voltage was $\sqrt{2} \times 5.78 = 8.16$ volts. Equating the maximum current of the voltage-phase cycle to the maximum E.M.F. divided by the impedance, we have

$$2.17 \times 10^{-3} = \frac{8.16}{\sqrt{363^2 + L^2\omega^2}};$$

whence

$$L\omega = 98,$$

and the angle of lag of the current of the voltage-phase cycle behind the external E.M.F. is

$$\varphi = \tan^{-1} \frac{98}{363} = 15.1°.$$

Coördinates. — The time coördinate is the abscissa of the oscillogram and is drawn as usual from left to right. The current values, which are the ordinates of the curve, are given by the scale at the left of the picture, one division of the scale being 10 milliamperes.

Discussion of the Oscillogram for Iron Pyrites. — Since a detailed discussion of the oscillograms taken with various crystals rectifiers has been given in Part II.,[1] the oscillogram of iron pyrites will be treated only briefly. Except for the slight "building up" segment of the photographic curve at rising from the zero axis, the upper loop of the curve is the same as that obtained with the substituted resistance, chosen to give an equal deflection. This means that the pyrites introduce no phase changes in this part of the cycle other than those introduced by a resistance. If now we examine the lower loop of the rectified cycle we find that its maximum leads the negative maximum of the voltage-phase cycle. The two-curves are also of very unequal amplitudes in their respective lower loops. Suppose we should have put into the circuit enough resistance to make their amplitudes the same in the negative loop, would the phase of their maxima have also been the same? This question

[1] See discussion of "building up" segment of Part II.

can be answered by a simple calculation. The amplitude of the negative loop of the cycle is one twelfth that of the voltage-phase cycle. Therefore, if R_1 is the resistance required to reduce the voltage-phase cycle to the amplitude of the negative maximum of the rectified cycle, we have

$$\frac{\sqrt{R_1^2 + 98^2}}{\sqrt{363^2 + 98^2}} = 12,$$

whence $R_1 = 4570$ ohms. The corresponding phase angle would be

$$\varphi_1 = 1.2°,$$

and the difference between φ and φ_1 would be 13.9°. The measured value of the lead of the negative maximum of the rectified cycle ahead of the voltage-phase cycle is 13.6°, which agrees with the calculated value within the limit of error of measuring the original photograph. Whence we may conclude that in the negative loop also the current through the rectifier has the same phase as it would have if replaced by a resistance producing the same amplitude.

Comparison of the Iron Pyrites Rectifier with Molybdenite. — In the present experiment a larger rectified current was obtained with the iron pyrites rectifier at practically perfect rectification than was obtained with the molybdenite rectifier. The apparent advantage of the pyrites rectifier arises in the employment of the deflecting coils of smaller impedance in the present experiment. The manner in which this affects the perfection of rectification was discussed above. A comparison of the current-voltage curves of Fig. 1 with the corresponding curves for molybdenite (Fig. 5, Part II.) shows that the molybdenite in the adjustment at which it was there used is a better rectifier than the iron pyrites at the adjustment at which it was used in the present experiment. The writer has tried several different adjustments of the two rectifiers with the result that the molybdenite was the more sensitive as a rectifier for alternating currents. It is entirely possible, however, that a comparison of other specimens of the two minerals may lead to the discovery of specimens of iron pyrites superior to the molybdenite. I should say that the two minerals have probably about equal capabilities as

detectors for electric waves, although up to the present I have not found the pyrites detector quite so easily adjusted or quite so rugged as the molybdenite detector.

We are apparently dealing with the same kind of a phenomenon in the case of all the crystalline substances used as detectors for electric waves. If we may accept the evidence obtained in the experiments on molydenite, described in Part II., this action does not take place through the intermediation of thermoelectricity. The fact that all the crystal rectifiers have also large thermoelectromotive forces against the common metals may be due to the fact that rectification and thermoelectricity have a common basis. For example, if we suppose that the surface of separation of two conductors has such a character as to permit the passage of electrons more easily in one direction than in the opposite direction, the boundary will act as a rectifier for alternating currents; it will also give rise to a thermoelectromotive force when heat is applied to the boundary, provided the velocity of the electrons is increased with an increase of temperature. This is merely given as a suggestion of one way in which the phenomena may be correlated. Further experiments are in progress.

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THE SOFTENING OF COLD, GLASS-HARD STEEL, IN TWENTY-FOUR YEARS.

By Laura C. Brant.

1. *Introductory.* — The electrical resistances and thermo-electric qualities of the following batch of about twenty glass-hard Stubb's steel rods (1 broken) were measured by Professor Barus[1] in June, 1885, only a few days after tempering, again in July, 1888, and finally in September, 1897. They were tempered glass-hard by heating with the electrical current and sudden quenching in cold water, by means of a specially designed machine, in a manner which insured marked uniformity in the results. The apparatus, the mass constants, homogeneity of the rods, etc., are fully given in the papers quoted. For the case of a given type of material like the steel examined, the electrical constants may be systematically computed. This facilitates many of the reductions. Thus the thermo-electric power y, in microvolts relatively to soft silver, bears to the specific resistance s_0 (microhms, c.c., 0° Centigrade) when both qualities vary with temper, the relation

$$y = 15.176 - 0.4123 s_0.$$

Moreover, the temperature coefficient a, corresponding to the specific resistance s_0, for corresponding variations is

$$(s_0 + .78)(a - .0001435) = .0682.$$

Steel is peculiarly favorable for observations of this kind, inasmuch as s_0 increases from soft to hard more than *threefold.* Hence the resistance variations due to temper in steel are enormous as compared with the corresponding variations due to drawn strains, etc., in the standards, so that this source of error is quite negligible. Naturally, the same standards (1 ohm and .1 ohm) were used throughout the work, these having been preserved together with the steel wires.

[1] Phil. Mag. (5), XXVI., p. 397, 1888; *ibid.* (5), XLIV., p. 486, 1897.

The following are the mean results for the electrical resistance as found in the earlier work (l. c.):

No. of Rods.	Diam.	Resistance, r_t, at $t°$ C., per cm.						Spec. Resistance, s_0, at 0° C.				
		1885	t	1888	t	1897	t	1885	1888	δs_0	1897	δs_0
	cm.											
Ten	.0806	9,042	27	8,232	28	7,717	22	45.0	40.7	−4.30	38.2	−6.73
Five	.044	30,536	25	27,892	29	26,112	22	44.8	40.6	−4.24	38.2	−6.64
Four	.1265	3,463	27	3,048	29	2,837	22	41.9	36.5	−5.49	34.2	−7.77

Comparing these results with the effect produced by annealing at higher temperatures (66°, 100°, 185°, etc.), Professor Barus concluded that atmospheric temperature acting for a period of 180 months had produced about the same effect as 66° acting for a period of 180 minutes. One may observe, moreover, that the thin rods fall off less and from higher initial tempers (resistances), while the thick rods fall off more and from smaller values.

2. *Data for May, 1909.* — The following results were obtained recently for the same steel rods and standards, by stepping off equal potentials from the ends of the rods and of the standards on the Wheatstone bridge. This makes it unnecessary to solder the ends, or of otherwise interfering with the rods, as the resistances of all connections, if fixed, are eliminated by the method. In the following table the individual resistances in microhms per linear cm. of length, at $t°$, the corresponding specific resistances at 0° Centigrade, as well as the mean values of these data are given in detail.

3. *Inferences.* — The results for specific resistance are graphically represented for the different diameters of steel wire in the figure.

Thus in the twenty-four years of observation the thinner rods have softened to the extent of losing nearly 20 per cent. (19.5 to 19.7) of the original specific resistance, while the thick rods have lost nearly 21 per cent. In both instances this diminution has taken place along the characteristic exponential curves discussed elsewhere. The total decrement from glass-hard to soft steel may be reckoned from 60 to 70 per cent., showing that a considerable part (about one third) of the maximum interval possible has been passed spontaneously at atmospheric temperatures. In fact the thick rods have nearly reached the practical annealing limit of 100°, acting for

Resistance per Linear cm., r_t, and Specific Resistance s_0, of Cold Glass-hard Steel, May, 1909. Effective Length of Rods about 23 cm.

No.	Diameter. $\times 10^3$	Resistance. r_t	Temperature. t	Specific Resistance. s_0 at 0° C.
	cm.	microhm.	° C.	
1	80.6	7,607	21.2	37.3
2		7,565	21.2	37.1
3		7,135	21.2	35.0
4		7,103	21.2	34.8
5		7,124	21.2	34.9
6		7,594	21.2	37.3
7		7,533	21.2	36.9
8		7,604	21.2	37.3
9		7,062	21.2	34.6
10		7,357	21.2	36.1
Mean.....	80.6	7,368	21.2	36.13
11	44	24,693	21.2	36.0
12		24,536	21.2	35.8
13		24,719	21.2	36.1
14		24,235	1.2	35.4
15		25,160	21.2	36.8
Mean.....	44	24,668	21.2	36.03
16	126.5	2,683	1.2	32.0
17		2,676	21.2	31.9
19		2,945	21.2	35.3
20		2,702	21.2	32.3
Mean.....	126.5	2,751	21.2	32.87

three hours, and they will probably quite reach this limit after forty years, *i. e.*, in 1925. In the thin rods the initial advantage of relatively greater hardness is sustained; in other words, the absolute amount of the decay has in both cases been about the same and equivalent to 8.8 to 9.1 microhms per c.c., respectively, during the twenty-four years.

The fact that the secular softening after about one quarter of a century already approaches the three hour annealing effect of 100°, suggests the interesting question whether in the lapse of sufficient time glass-hard steel would actually reach the soft and plastic states spontaneously. If the mean rates for the last twelve years be taken in case of the above steels, the thin rods should be soft in 170 years and the thick rods in 250 years, results of course which are

inferior limits of the interval in question. It does not seem improbable, however, that if n be the number of hard molecules per c.c. of steel, the rate $-dn/dt$, where t is the time of exposure to a given temperature, will be some function of n. The change there-

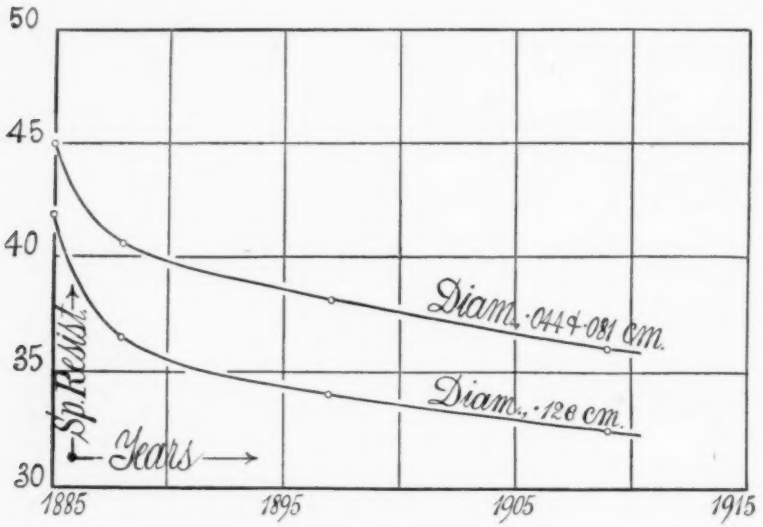

Fig. 1.

fore will take place at a diminishing rate through infinite time, but in a way that the eventual complete softening of hard steel after a period great in comparison with 250 years is altogether probable.

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